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# Pion, Kaon, Proton and Antiproton Production in Proton-Proton Collisions

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October 2008

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## Abstract

*Inclusive pion, kaon, proton, and antiproton production from proton-proton collisions is studied at a variety of proton energies. Various available parameterizations of Lorentz-invariant differential cross sections as a function of transverse momentum and rapidity are compared with experimental data. The Badhwar and Alper parameterizations are moderately satisfactory for charged pion production. The Badhwar parameterization provides the best fit for charged kaon production. For proton production, the Alper parameterization is best, and for antiproton production the Carey parameterization works best. However, no parameterization is able to fully account for all the data.*

## 1 Introduction

The peak of the galactic cosmic ray spectrum occurs right near the pion production threshold. Therefore, it is important to include pion and other particle production cross sections in space radiation transport codes. A widely used code is HZETRN [1, 2]. This code has been modified to include pion production [3]. A current goal is to include the production and propagation of all important hadronic species. In order to do this, one needs cross sections for production of all important particles. The particle cross sections in space radiation transport codes need to be accurate in the intermediate energy region of a few GeV where the cosmic ray spectrum peaks. In addition, a current goal is to produce a fully three dimensional version of HZETRN, which therefore requires three dimensional differential cross sections. The aim of the present paper is to test currently available parameterized cross sections for kaon and antiproton production, and to see if they are suitable for use in space radiation codes. Pion and proton cross sections were also available and these are also included here for completeness.

## 2 Kinematics

Consider the *inclusive* reaction

$$a + b \rightarrow c + X , \tag{1}$$

where  $c$  is the produced particle of interest and  $X$  is anything. Throughout this paper we assume that all variables, such as momentum, are evaluated in the center of momentum (cm) frame, unless otherwise indicated. Lab frame variables will be given a subscript. For example, the variable  $x$  evaluated in the cm frame is written as  $x$  but evaluated in the lab frame it is written as  $x_{\text{lab}}$ . The momentum of particle  $c$  is denoted as  $p$ , and supposing that it scatters at angle  $\theta$  to the beam direction, then the longitudinal and transverse

components of momentum are

$$p_z \equiv p \cos \theta , \quad (2)$$

$$p_T \equiv p \sin \theta . \quad (3)$$

Note that

$$p^2 = p_z^2 + p_T^2 , \quad (4)$$

$$\tan \theta = p_T / p_z . \quad (5)$$

Feynman used a scaled variable instead of  $p_z$  itself [4, 5, 6, 7]. The *Feynman scaling variable* is [6, 8, 9, 10, 11, 12, 13]

$$x_F \equiv \frac{p_z}{p_{z \max}} , \quad (6)$$

where  $p_z$  is the longitudinal momentum of the produced meson in the cm frame, and  $p_{z \max}$  is the maximum transferable momentum given by [9, 10, 13]

$$p_{z \max} = \sqrt{\frac{\lambda(s, m_c, m_X)}{4s}} , \quad (7)$$

with

$$\lambda(s, m_i, m_j) \equiv (s - m_i^2 - m_j^2)^2 - 4m_i^2 m_j^2 . \quad (8)$$

Note that

$$p_{z \max} = p_{\max} . \quad (9)$$

Nagamiya and Gyulassy [10] point out that if  $c$  is a boson with zero baryon number, then

$$m_X = m_A + m_B , \quad (10)$$

in agreement with the  $p_{z \max}$  formulas of Nagamiya and Gyulassy [10] and Cassing [13]. The Feynman scaling variable approaches the limiting value [11]

$$x_F \rightarrow \frac{2p_z}{\sqrt{s}} , \quad \text{as } s \rightarrow \infty . \quad (11)$$

Also, it is obviously bounded in the following manner [6]

$$-1 < x_F < 1 . \quad (12)$$

Sets of variables that are often used are either  $(p, \theta)$  or  $(p_z, p_T)$ . Writing

$$p_z = x_F \sqrt{\frac{\lambda(s, m_c, m_X)}{4s}} \quad (13)$$

shows that another useful and common variable set is  $(x_F, p_T)$ , which is used by Alt *et al.* [14, 15] when presenting their data. These variables are also used throughout the present work. Rapidity is defined as

$$y \equiv \frac{1}{2} \log \left( \frac{E + p_z}{E - p_z} \right) , \quad (14)$$

so that

$$E = m_T \cosh y , \quad (15)$$

$$p_z = m_T \sinh y , \quad (16)$$

where the transverse mass is defined through

$$m_T^2 \equiv m^2 + p_T^2 = E^2 - p_z^2 , \quad (17)$$

with  $m$  as the mass of the produced particle  $c$ . This gives yet another useful variable set  $(y, p_T)$ . In the following work, it will be necessary to write the rapidity in terms of the Feynman scaling variable as

$$y = \frac{1}{2} \log \left( \frac{\sqrt{x_F^2 + m_T^2/p_{z\max}^2} + x_F}{\sqrt{x_F^2 + m_T^2/p_{z\max}^2} - x_F} \right) . \quad (18)$$

### 3 Parameterizations

Blattnig *et al.* [16, 17] did a study of the various parameterizations available for inclusive pion production in proton-proton collisions. They concluded that the Badhwar parameterization [18] worked the best for charged pion production. However, other parameterizations [16, 19, 20, 21, 22] will be reviewed again to see which works best for the new experimental data. The Alt *et al.* [14, 15] data set uses the variables  $(x_F, p_T)$ , whereas some of the other parameterizations are written in terms of other variables sets. These will need to be converted to  $(x_F, p_T)$ .

### 3.1 Badhwar parameterization

The Badhwar parameterization [18] gives the Lorentz-invariant differential cross section for charged pions as

$$E \frac{d^3\sigma}{d^3p}(\pi^\pm) = \frac{A}{(1 + 4m_p^2/s)^r} (1 - \tilde{x})^q \exp\left[\frac{-Bp_T}{1 + 4m_p^2/s}\right], \quad (19)$$

and neutral pions as

$$E \frac{d^3\sigma}{d^3p}(\pi^0) = Af(E_p)(1 - \tilde{x})^q \exp\left[\frac{-Bp_T}{1 + 4m_p^2/s}\right], \quad (20)$$

and charged kaons as

$$E \frac{d^3\sigma}{d^3p}(K^\pm) = A(1 - \tilde{x})^C \exp(-Bp_T), \quad (21)$$

where  $m_p$  is the proton mass,  $\sqrt{s}$  is the total energy in the center of momentum (cm) frame, and  $p_T$  is the transverse momentum of the produced meson in the cm frame. The other terms are given by

$$\tilde{x} = \left[ x_F^2 + \frac{4}{s}(p_T^2 + m^2) \right]^{1/2}, \quad (22)$$

where it is assumed that the variables appearing in  $x_F$  are in the cm frame. The mass  $m$  is the mass of the produced particle (pion or kaon). Badhwar writes  $x_\parallel^* \equiv x_F$ . Also,

$$q = \frac{C_1 + C_2 p_T + C_3 p_T^2}{\sqrt{1 + 4m_p^2/s}}. \quad (23)$$

The function  $f(E_p)$  for neutral pions is given by

$$f(E_p) = (1 + 23E_p^{-2.6})(1 - 4m_p^2/s)^r, \quad (24)$$

with the constants listed in Table 1. Badhwar points out that for large values of  $E_p$ , equation (20) takes the asymptotic form

$$E \frac{d^3\sigma}{d^3p}(\pi^0) = A \exp(-Bp_T)(1 - \tilde{x})^{(C_1 - C_2 p_T + C_3 p_T^2)}, \quad (25)$$

consistent with the Feynman scaling hypothesis [6]. The Badhwar variables are  $(x_F, p_T)$ , which are also used in the Alt *et al.* [14, 15] data, and no variable conversion is necessary.

Table 1: Constants for the Badhwar parameterization.

Particle	$A$	$B$	$r$	$C$	$C_1$	$C_2$	$C_3$
$\pi^+$	153	5.55	1	$\dots$	5.3667	-3.5	0.8334
$\pi^-$	127	5.3	3	$\dots$	7.0334	-4.5	1.667
$\pi^0$	140	5.43	2	$\dots$	6.1	3.3	0.6
$K^+$	8.85	4.05	$\dots$	2.5	$\dots$	$\dots$	$\dots$
$K^-$	9.3	3.8	$\dots$	8.3	$\dots$	$\dots$	$\dots$

### 3.2 Alper parameterization

The Alper [19] parameterization for charged pions and kaons, protons and antiprotons is

$$E \frac{d^3\sigma}{d^3p} = A_1 \exp(-Bp_T) \exp(-Dy^2) + A_2 \frac{(1 - p_T/p_{\text{beam}})^m}{(p_T^2 + M^2)^n}, \quad (26)$$

with the constants listed in Table 2. The Alper variables are  $(y, p_T)$ . To change to the variables  $(x_F, p_T)$ , we convert the rapidity in equation (26) to  $x_F$  using equation (18).

Table 2: Constants for the Alper parameterization.

Particle	$A_1$	$B$	$D$	$A_2$	$M$	$m$	$n$
$\pi^+$	210	7.58	0.20	10.7	1.03	10.9	4.0
$\pi^-$	205	7.44	0.21	12.8	1.08	13.1	4.0
$K^+$	14.3	6.78	1.5	8.0	1.29	12.1	4.0
$K^-$	13.4	6.51	1.8	9.8	1.39	17.4	4.0
$p$	5.3	3.8	-0.2	16	1.2	0	7.5
$\bar{p}$	1.89	4.1	2.3	25	1.41	25	4.5

### 3.3 Ellis parameterization

The Ellis [20] parameterization for charged pions, neutral pions, charged kaons, protons and antiprotons is

$$E \frac{d^3\sigma}{d^3p} = A(p_T^2 + M^2)^{-N/2} (1 - x_T)^F, \quad (27)$$

where  $A$  is an overall normalization fitted to be  $A = 13$  in reference [16] and  $x_T \equiv p_T/p_{\max} \approx 2p_T/\sqrt{s}$ . The same value of  $A$  is used in the present work. The other constants are listed in Table 3. The Ellis parameterization is independent of the emission angle  $\theta$ , and so does not carry any dependence on  $p_z$ ,  $x_F$ ,  $y$  etc.

Table 3: Constants for the Ellis parameterization.

Particle	$N$	$M^2$	$F$
$\pi^+$	7.70	0.74	11.0
$\pi^-$	7.78	0.79	11.9
$\pi^0$	10.8	2.3	7.1
$K^+$	8.72	1.69	9.0
$K^-$	8.76	1.77	12.2
$p$	10.38	1.82	7.3
$\bar{p}$	9.1	1.17	14.0

### 3.4 Mokhov parameterization

The Mokhov [21] parameterization is

$$E \frac{d^3\sigma}{d^3p} = A \left(1 - \frac{p}{p_{\max}}\right)^B \exp\left(-\frac{p}{C\sqrt{s}}\right) V_1(p_T) V_2(p_T), \quad (28)$$

where

$$\begin{aligned} V_1(p_T) &= (1 - D) \exp(-E p_T^2) + D \exp(-F p_T^2) && \text{for } p_T \leq 0.933 \text{ GeV}, \\ V_1(p_T) &= \frac{0.2625}{(p_T^2 + 0.87)^4} && \text{for } p_T > 0.933 \text{ GeV}, \end{aligned} \quad (29)$$

and

$$\begin{aligned} V_2(p_T) &= 0.7363 \exp(0.875 p_T) && \text{for } p_T \leq 0.35 \text{ GeV}, \\ V_2(p_T) &= 1 && \text{for } p_T > 0.35 \text{ GeV}, \end{aligned} \quad (30)$$

with the constants listed in Table 4. Using  $p = \sqrt{p_z^2 + p_T^2}$ , gives the Mikhov variables  $(p_z, p_T)$  which are transformed to  $(x_F, p_T)$  using equation (13).

Table 4: Constants for the Mikhov parameterization.

Particle	$A$	$B$	$C$	$D$	$E$	$F$
$\pi^+$	60.1	1.9	0.18	0.3	12	2.7
$\pi^-$	51.2	2.6	0.17	0.3	12	2.7

### 3.5 Carey parameterization

The Carey [22] parameterization, for negative pions, negative kaons, and antiprotons is

$$E \frac{d^3\sigma}{d^3p} = hN(p_T^2 + G)^{-4.5} (1 - x_R)^J, \quad (31)$$

where  $N$  is an overall normalization fitted to be  $N = 13$  in reference [16] and  $x_R \equiv p/p_{\max} \approx 2p/\sqrt{s}$ . The same value of  $N$  is used in the present work. The constants are listed in Table 5. The Carey variables are  $(p_z, p_T)$ . To change to the variables  $(x_F, p_T)$ , we use  $x_R = \sqrt{x_F^2 + p_T^2}/p_{\max}$ .

Table 5: Constants for the Carey parameterization.

Particle	$N$	$h$	$G$	$J$
$\pi^-$	13	1.0	0.86	4
$K^-$	13	0.36	1.22	5
$\bar{p}$	13	0.26	1.04	7

## 4 Comparison to experiment

The various parameterizations are compared to the experimental results of Alper *et al.* [19] in figures 1 - 33. We now discuss how well they agree.

### 4.1 Pions

Pion results are shown in figures 1 - 14. All fits are of similar quality when comparing  $\pi^+$  to  $\pi^-$ . The Carey parameterization only applies to  $\pi^-$ . The Badhwar and Alper parameterizations provide an excellent fit to the data for low values of transverse momentum  $p_T$ , but fail for high  $p_T$ , with the Badhwar parameterization underpredicting data at high  $p_T$  and the Alper parameterization overpredicting at high  $p_T$ . The Ellis and Carey parameterizations work well at high  $p_T$  but fail at low  $p_T$ . The Mokhov parameterization is the poorest. It does not work well in any  $p_T$  region. None of the parameterizations work well for all values of  $p_T$ . *For space radiation purposes, where large cross section values are the most important and which occur in the low  $p_T$  region, we conclude that either the Badhwar or Alper parameterization would be moderately satisfactory.* Further work is needed to provide a good fit for all  $p_T$  values.

### 4.2 Kaons

Kaon results are shown in figures 15 - 25. All fits are of similar quality when comparing  $K^+$  to  $K^-$ . The Carey parameterization only applies to  $K^-$ . Comparison between the various parameterizations and experiment is similar to the pion case. (But there is no Mokhov parameterization.) However, *here the Badhwar parameterization is clearly superior to all the others.*

### 4.3 Proton and antiproton

Proton and antiproton results are shown in figures 26 - 33. Unlike the pion and kaon case, the fits here are of different quality depending on whether the particle is a proton or antiproton. The Badhwar parameterization is not available for protons and antiprotons. The Carey parameterization only applies to antiprotons. The Alper parameterization for protons is far superior to the Ellis parameterization. However, the Alper and Ellis results for antiprotons are poor. The Carey results for antiprotons are quite good. *It is recommended that the Alper parameterization be used for protons and the Carey parameterization be used for antiprotons.*



## 5 Conclusions

Inclusive production of pions, kaons, protons and antiprotons has been studied in proton-proton collisions for incident proton energies of  $\sqrt{s} = 23, 31, 45, 53$ , and  $63$  GeV, corresponding to incident lab kinetic energies of  $T_{\text{lab}} = 280, 510, 1077, 1495$  and  $2113$  GeV, respectively. Various available parameterizations have been compared to the experimental data of Alper *et al.* [19]. The Badhwar or Alper parameterizations are moderately satisfactory for charged pion production. The Badhwar parameterization provides the best fit for charged kaon production. For proton production, the Alper parameterization is best, and for antiproton production the Carey parameterization works best. There is no parameterization available that works well for all particles at all values of  $p_T$ . Further work is needed to improve this situation, as well as studying lower energy.

Based on the recommendations of the previous section, it is appropriate to include some of these parameterizations into modifications of HZETRN when it is upgraded to perform hadron transport. The Badhwar or Alper parameterization will be used for pion production, the Badhwar parameterization will be used for kaon transport, the Alper parameterization will be used for proton transport and the Carey parameterization will be used for antiproton transport. These parameterizations will be adequate for a first approximation. A better transport methodology will include improvements at lower energies.

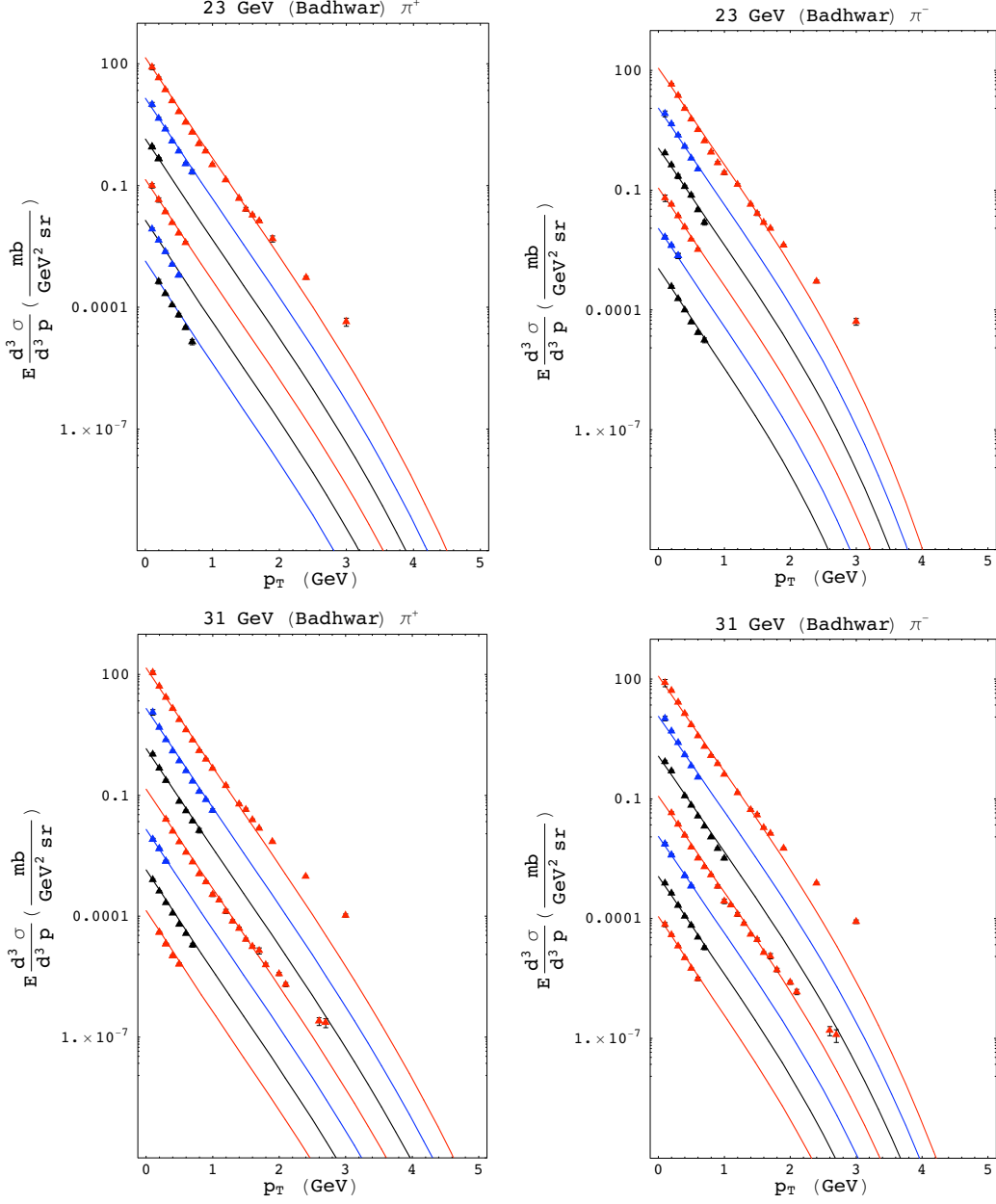


Figure 1: Badhwar parameterization versus experiment [19] for inclusive  $\pi^+$  and  $\pi^-$  production in  $pp$  collisions at  $\sqrt{s} = 23$  GeV and  $\sqrt{s} = 31$  GeV. The rapidity for all of the top curves in each frame is  $y = 0.0$ . It increases in steps of 0.2 from the top to the bottom curves in each frame. The data and lines in each frame are multiplied successively by 0.1 to allow for a better separation.

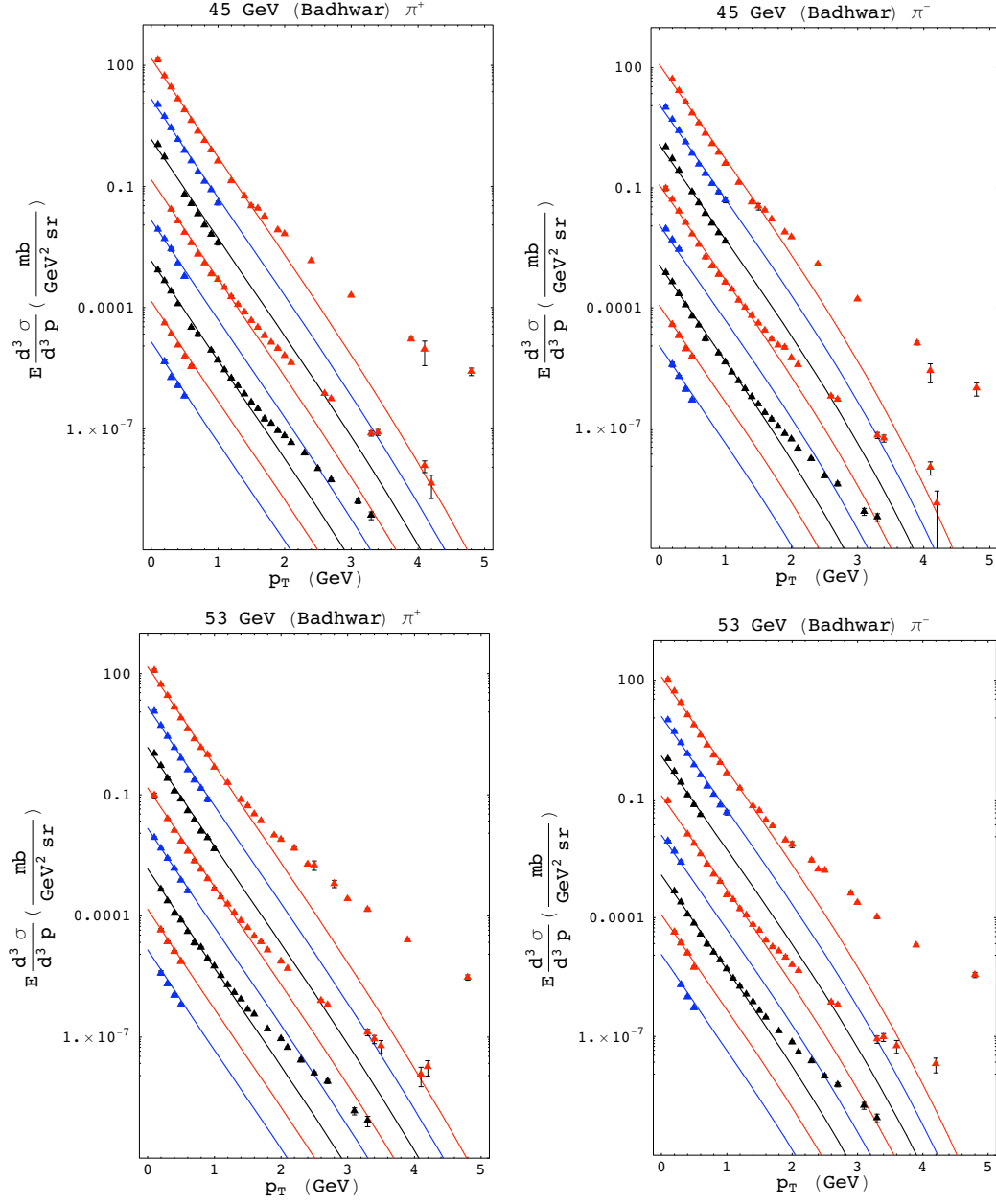


Figure 2: Same as figure 1, except that  $\sqrt{s} = 45$  GeV and  $\sqrt{s} = 53$  GeV.

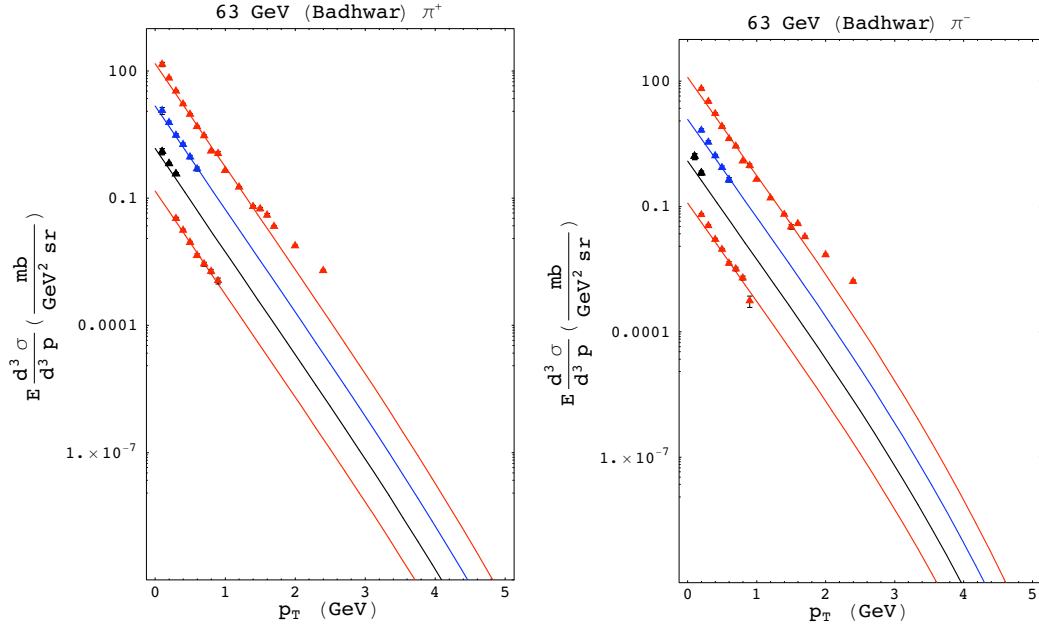


Figure 3: Same as figure 1, except that  $\sqrt{s} = 63$  GeV.

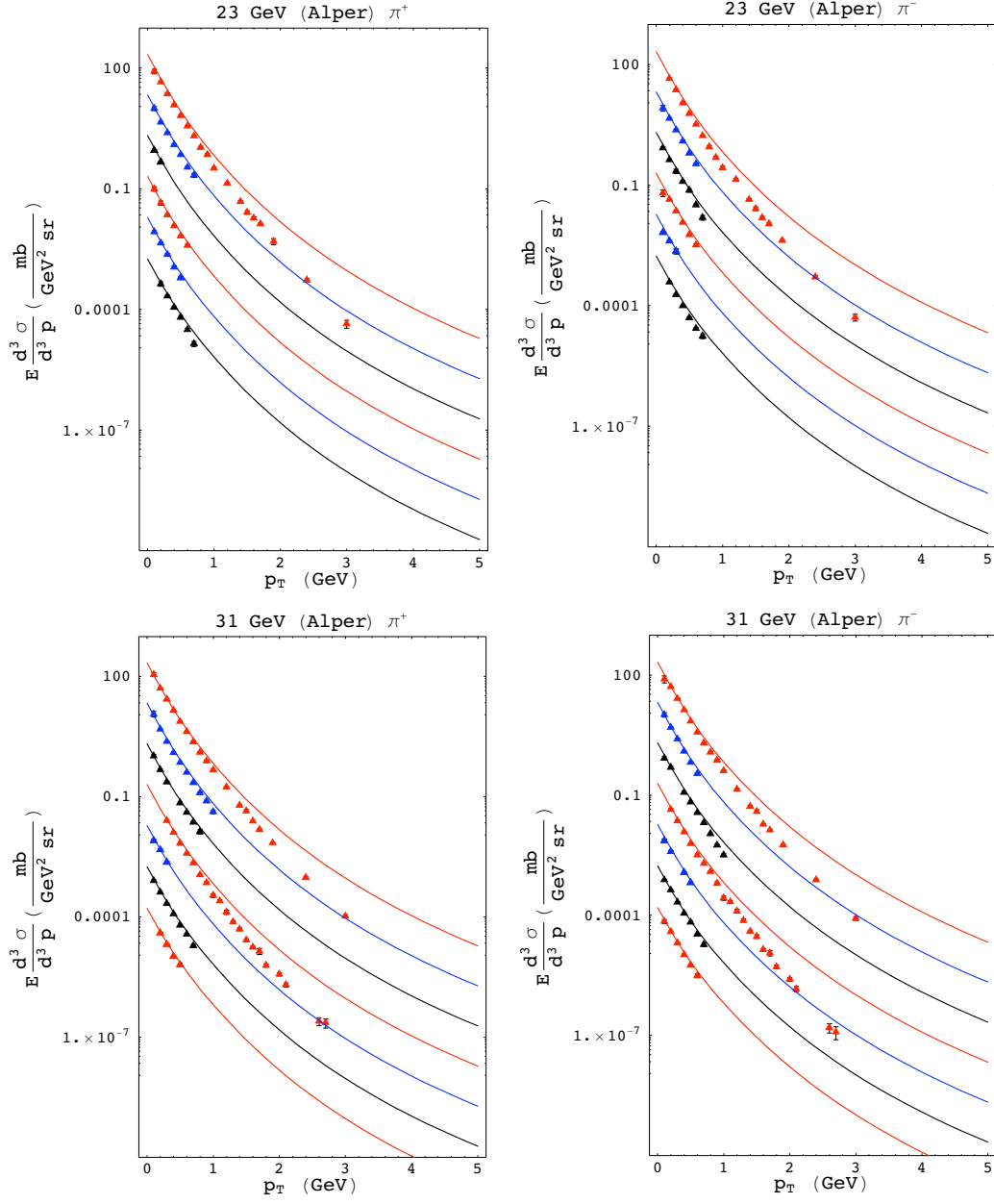


Figure 4: Same as figure 1, except with Alper parameterization.

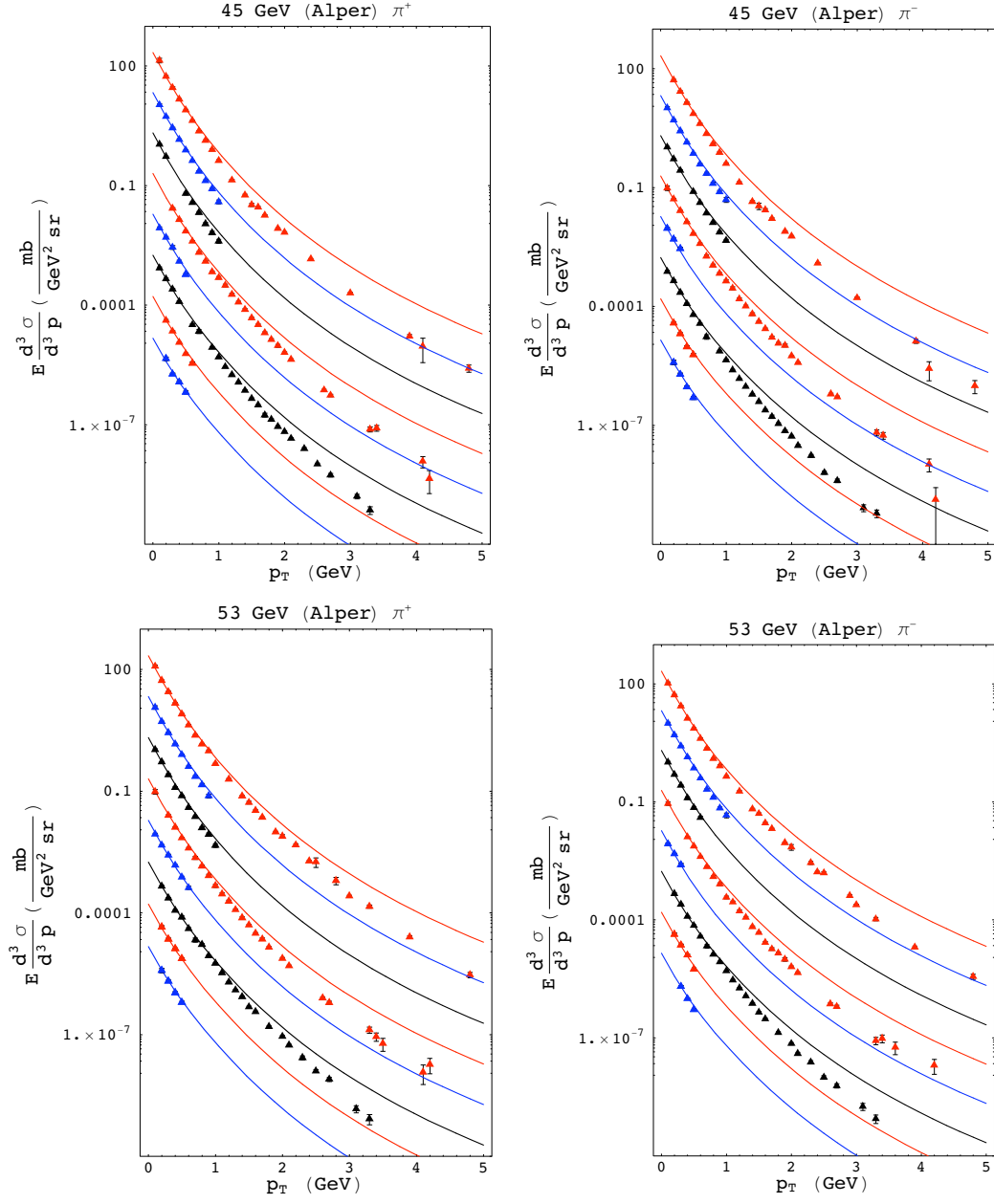


Figure 5: Same as figure 2, except with Alper parameterization.

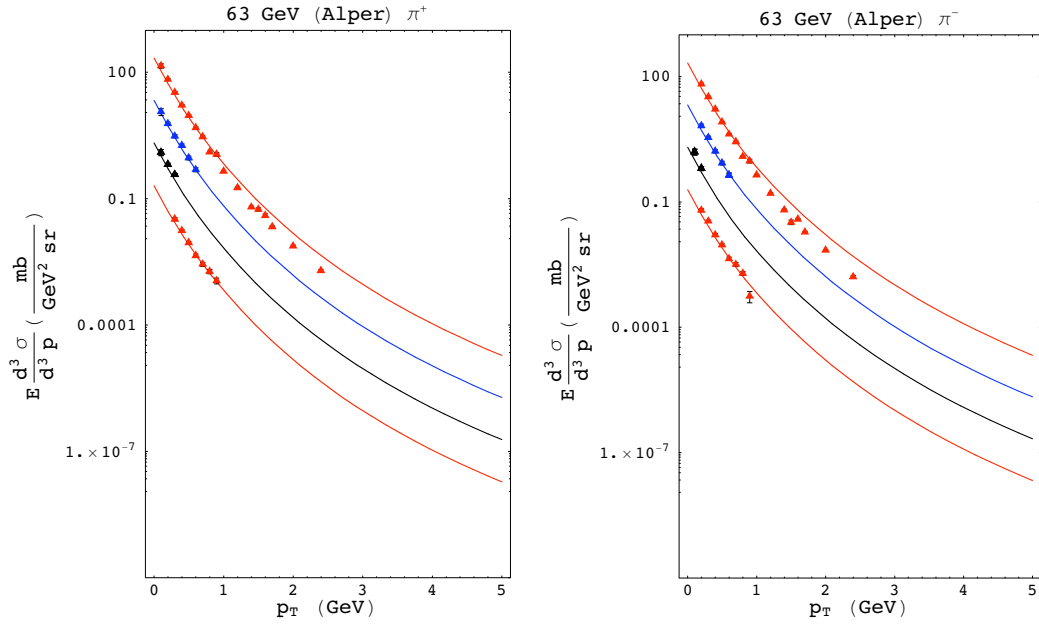


Figure 6: Same as figure 3, except with Alper parameterization.

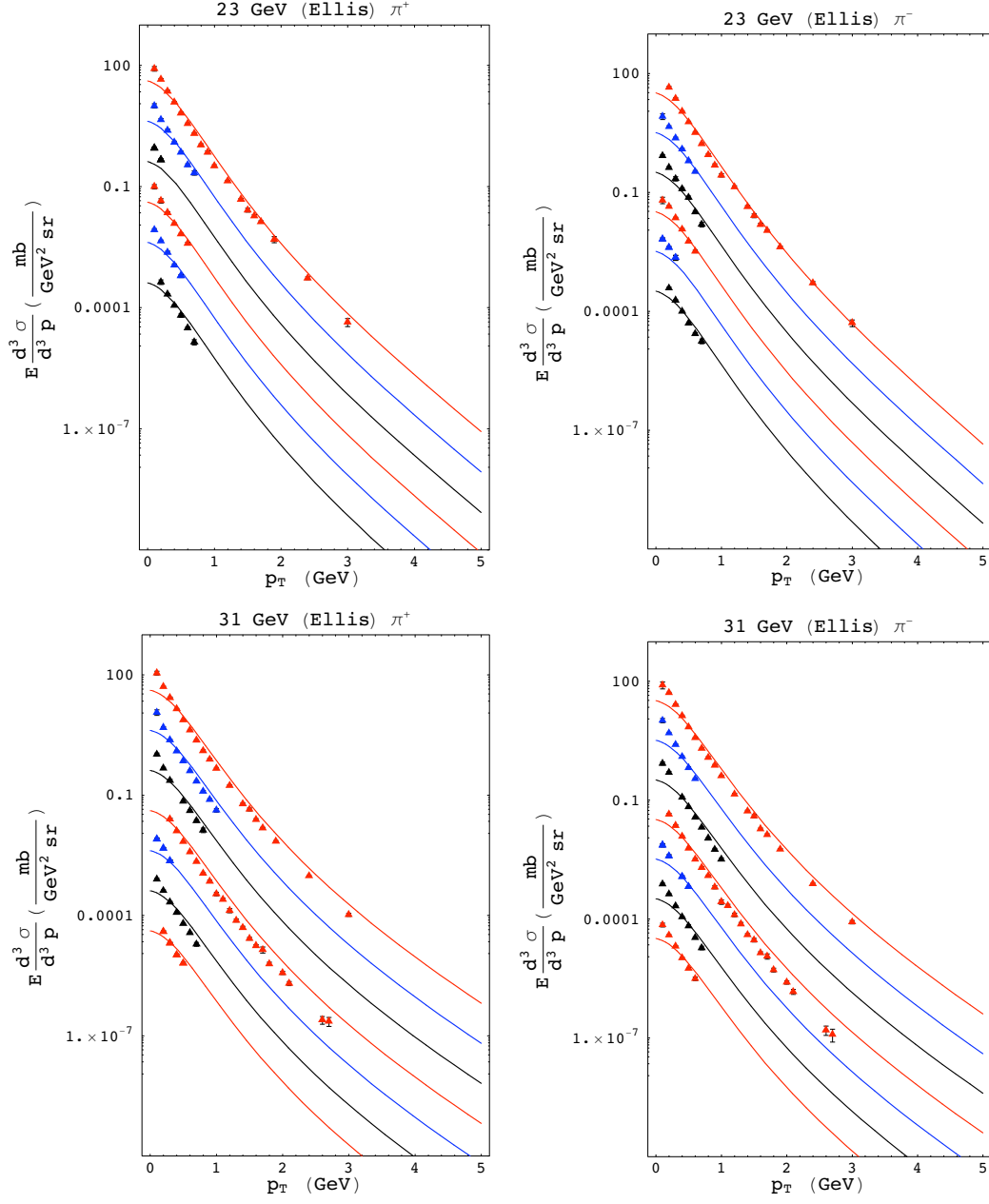


Figure 7: Same as figure 1, except with Ellis parameterization.



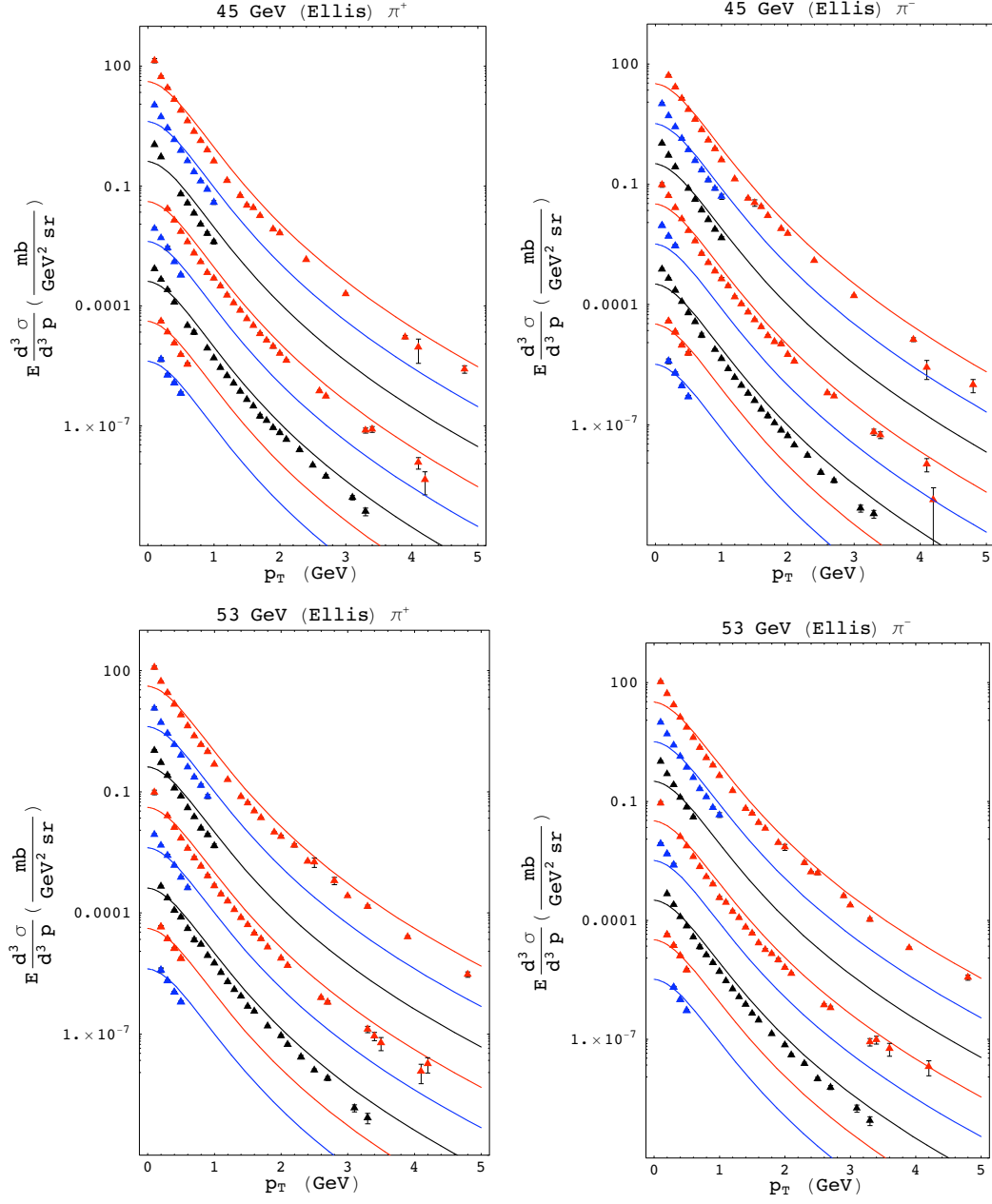


Figure 8: Same as figure 2, except with Ellis parameterization.

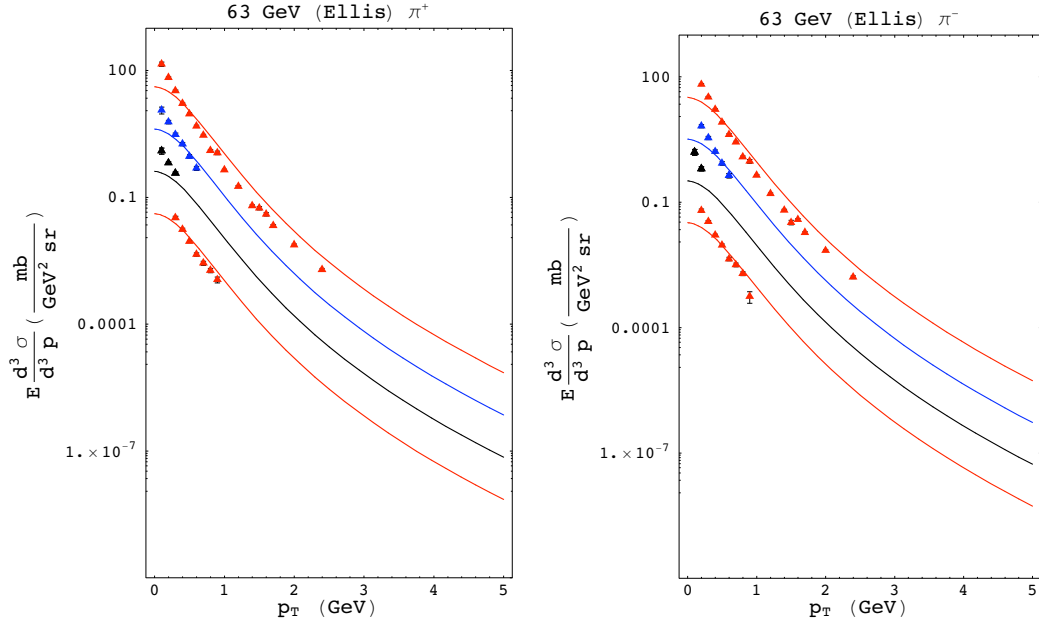


Figure 9: Same as figure 3, except with Ellis parameterization.

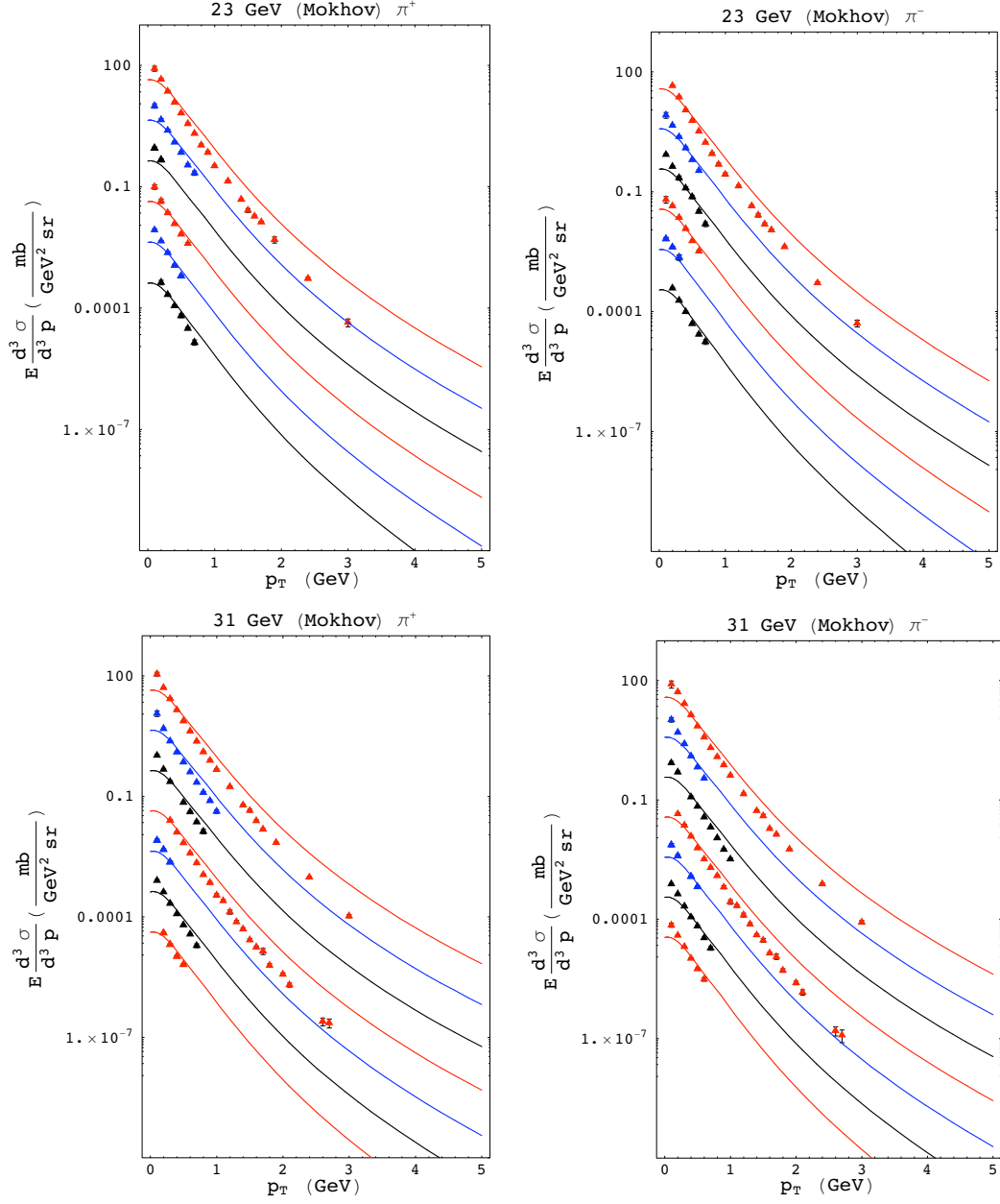


Figure 10: Same as figure 1, except with Mokhov parameterization.

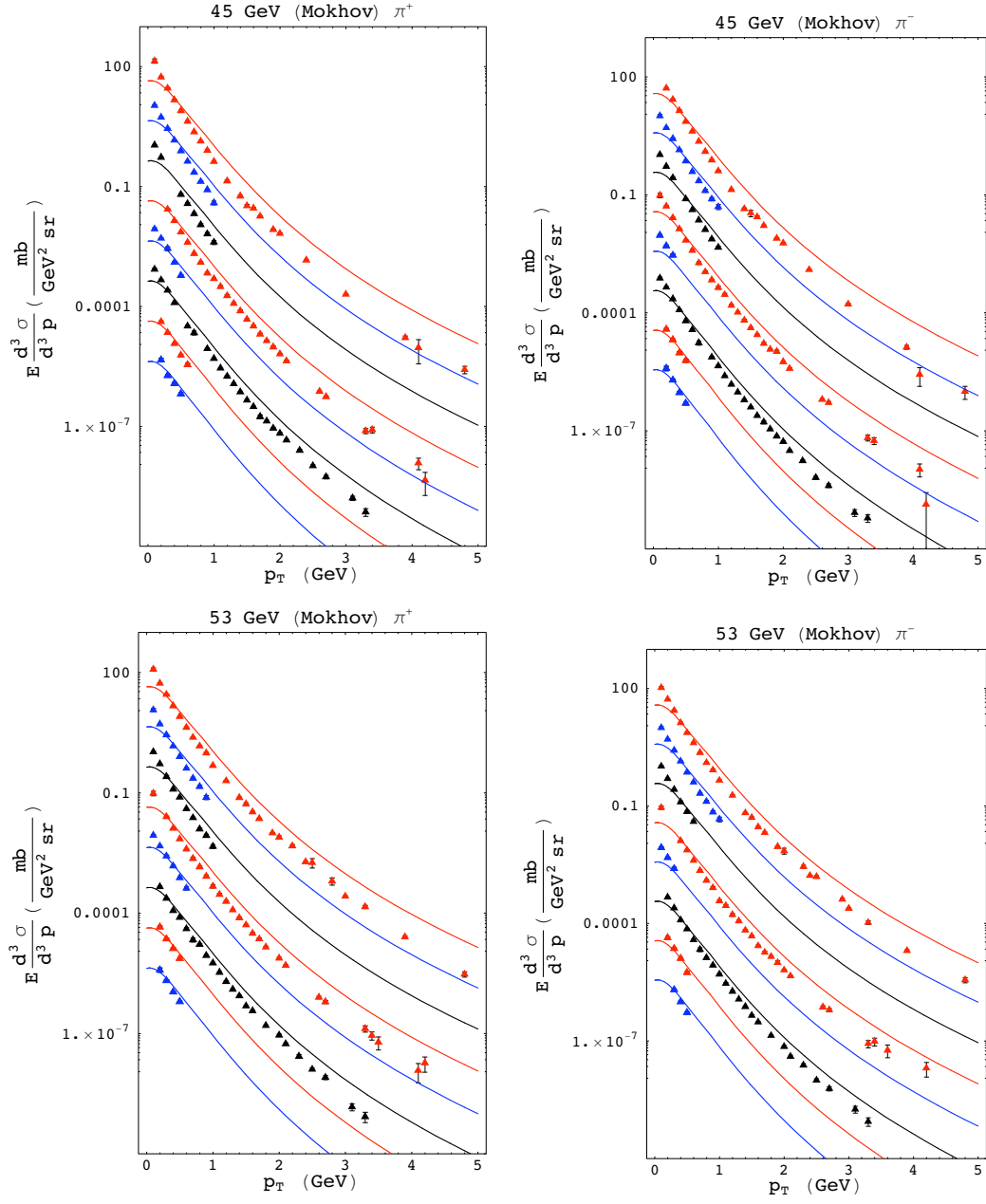


Figure 11: Same as figure 2, except with Mikhov parameterization.

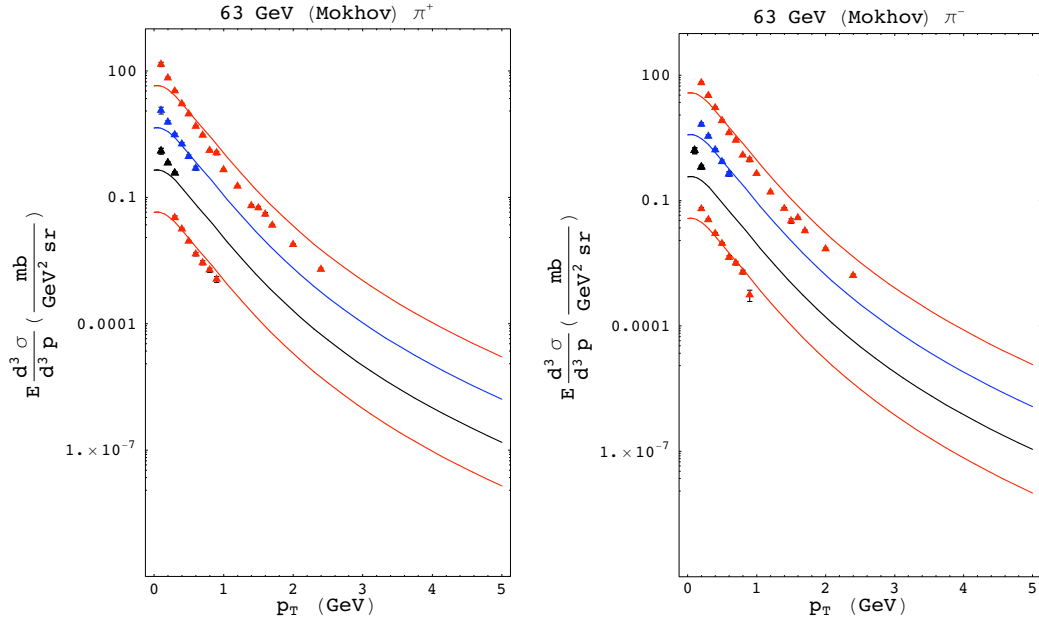


Figure 12: Same as figure 3, except with Mokhov parameterization.

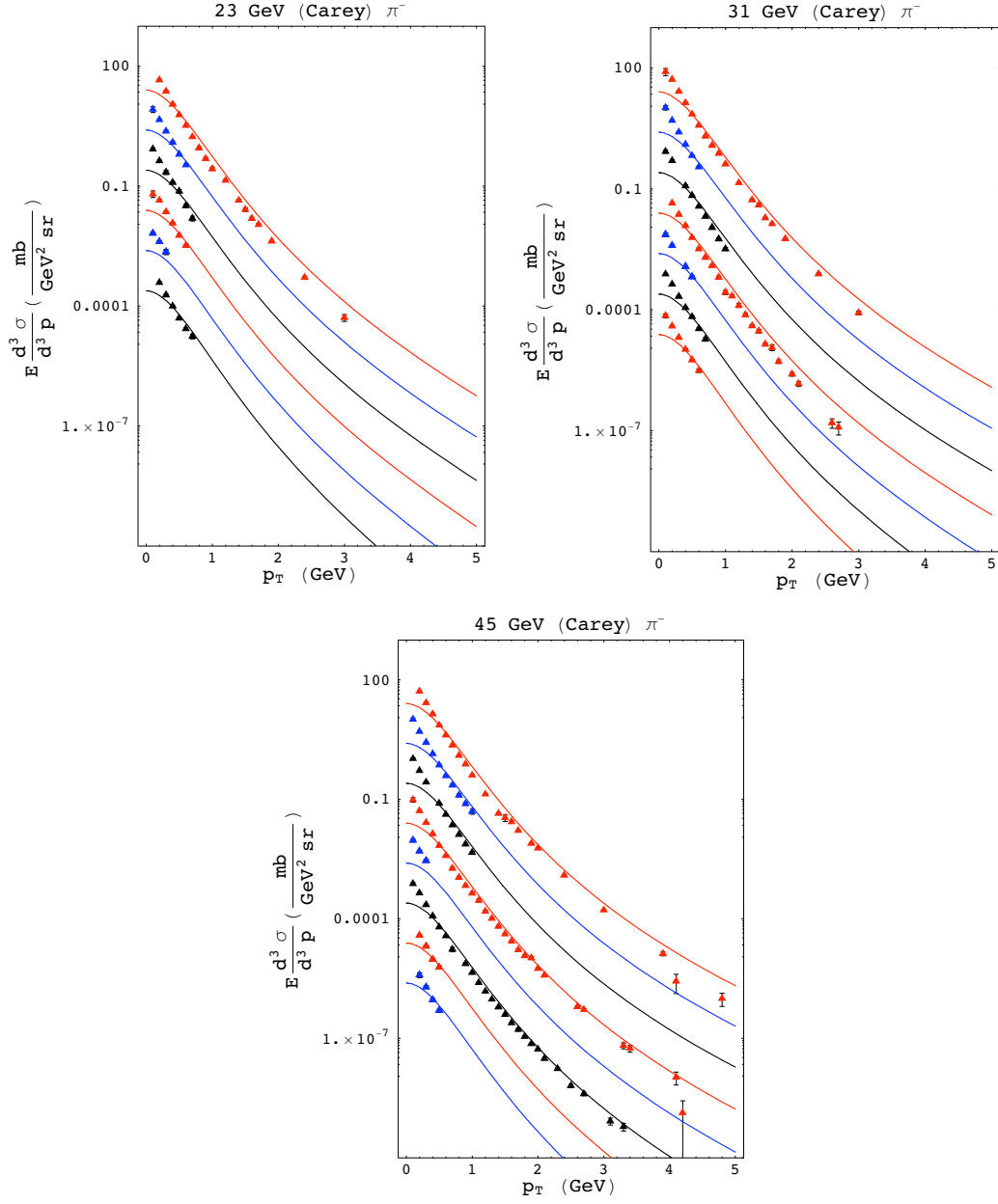


Figure 13: Same as figure 1, except with Carey parameterization with  $\sqrt{s} = 23$  GeV and  $\sqrt{s} = 31$  GeV and  $\sqrt{s} = 45$  GeV for  $\pi^-$  production only.

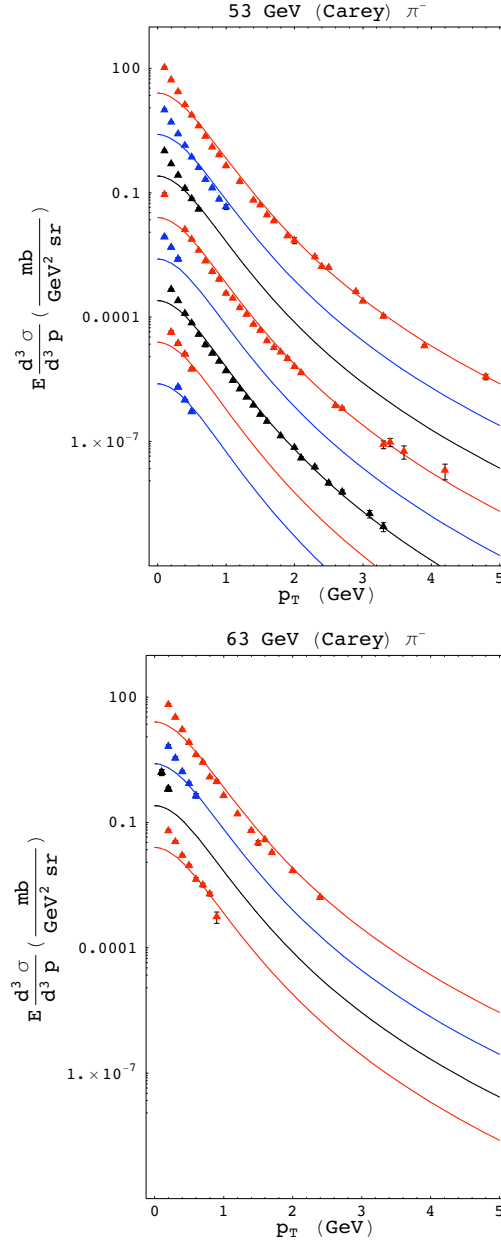


Figure 14: Same as figure 1, except with Carey parameterization with  $\sqrt{s} = 53$  GeV and  $\sqrt{s} = 63$  GeV for  $\pi^-$  production only.

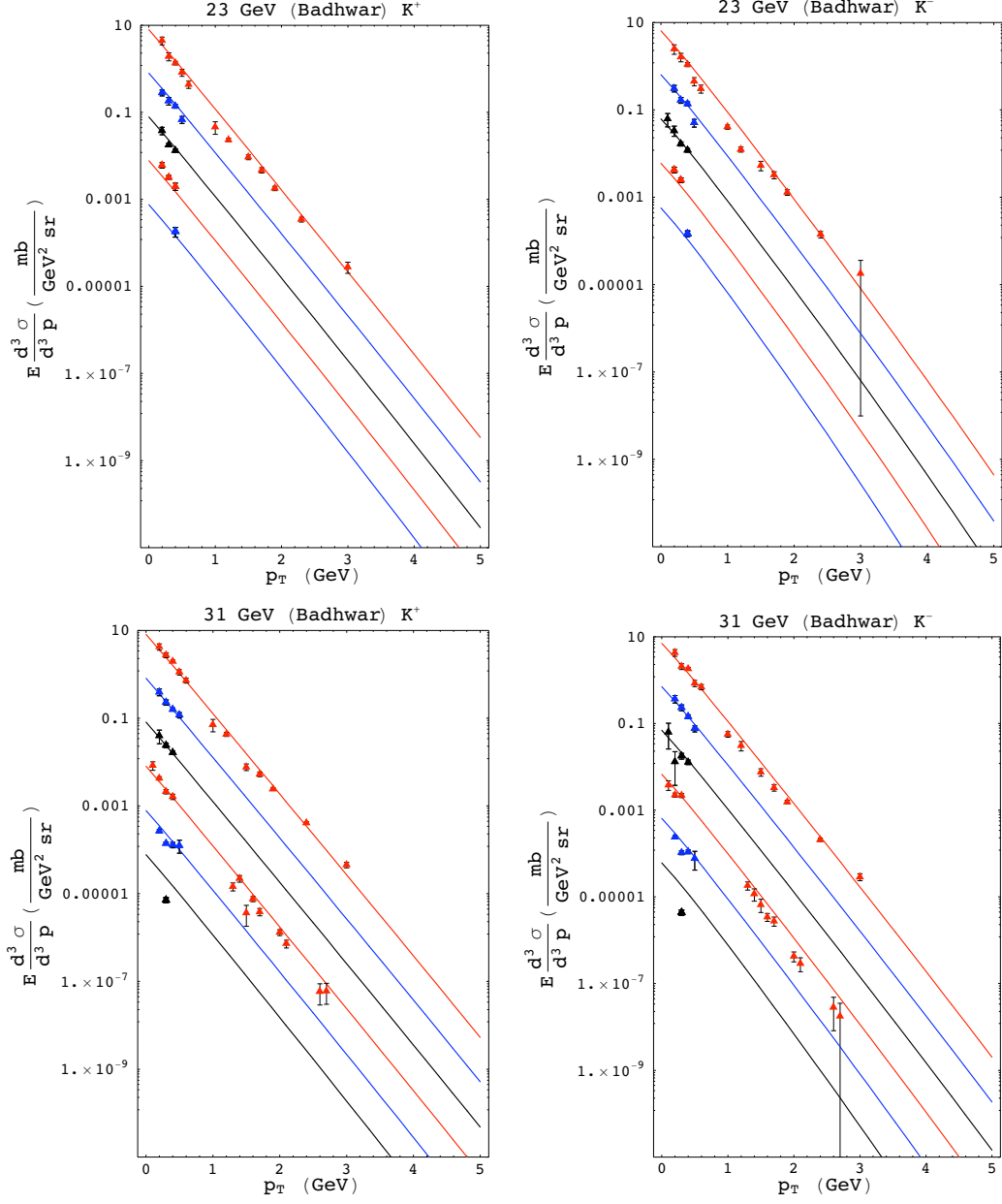


Figure 15: Badhwar parameterization versus experiment [19] for inclusive  $K^+$  and  $K^-$  production in  $pp$  collisions at  $\sqrt{s} = 23$  GeV and  $\sqrt{s} = 31$  GeV. The rapidity for all of the top curves in each frame is  $y = 0.0$ . It increases in steps of 0.2 from the top to the bottom curves in each frame. The data and lines in each frame are multiplied successively by 0.1 to allow for a better separation.



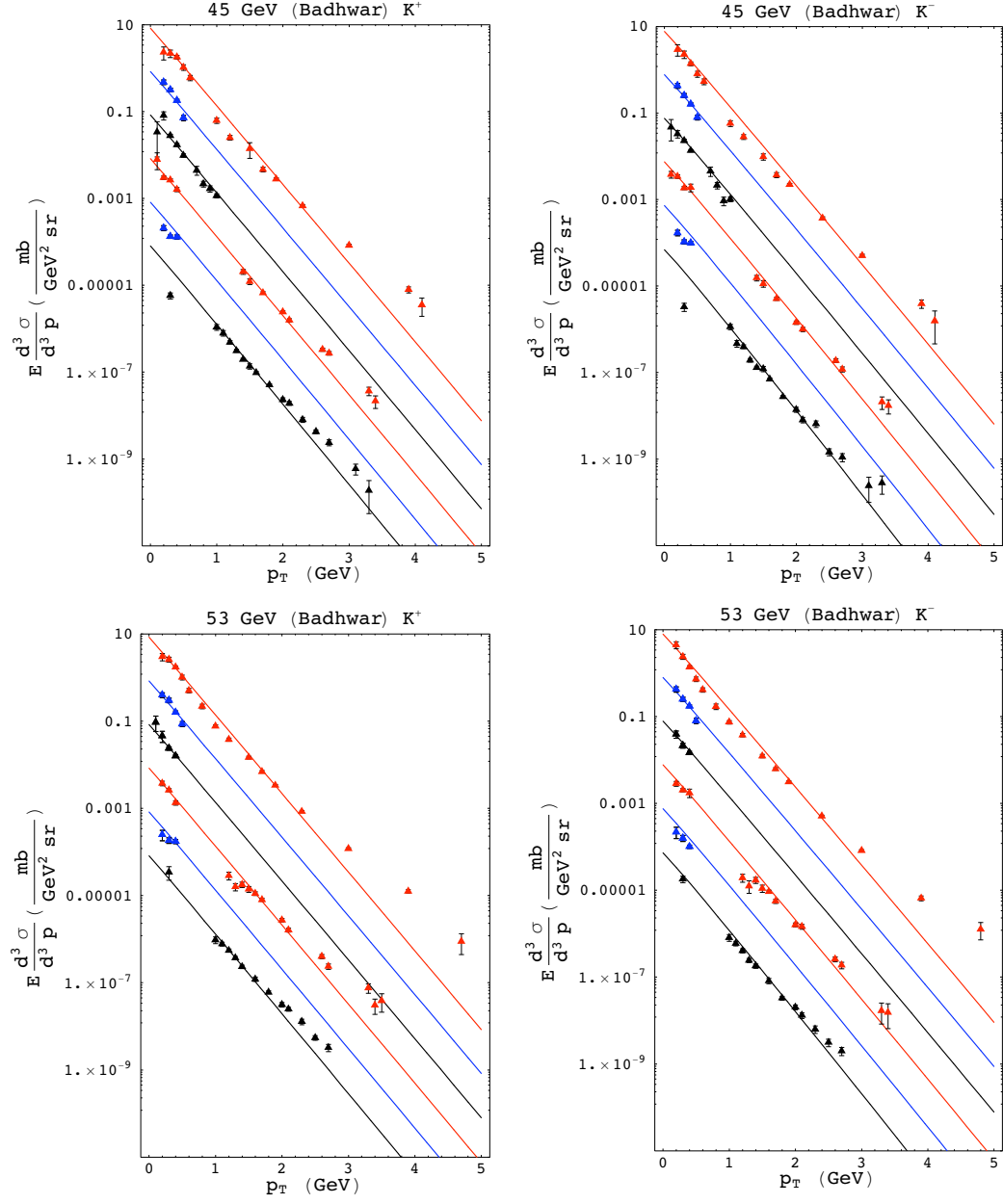


Figure 16: Same as figure 15, except that  $\sqrt{s} = 45$  GeV and  $\sqrt{s} = 53$  GeV.

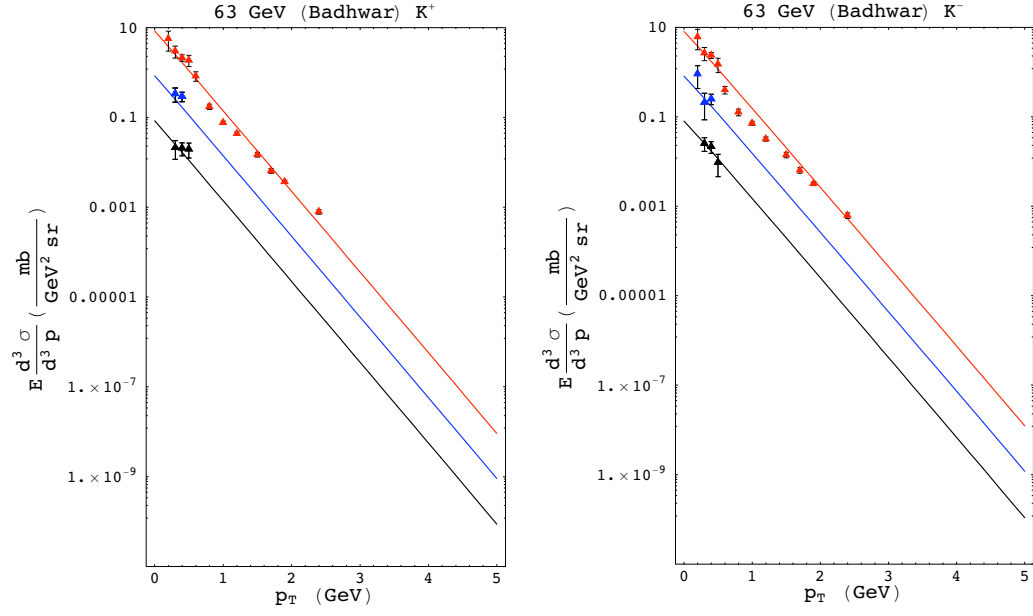


Figure 17: Same as figure 15, except that  $\sqrt{s} = 63$  GeV.

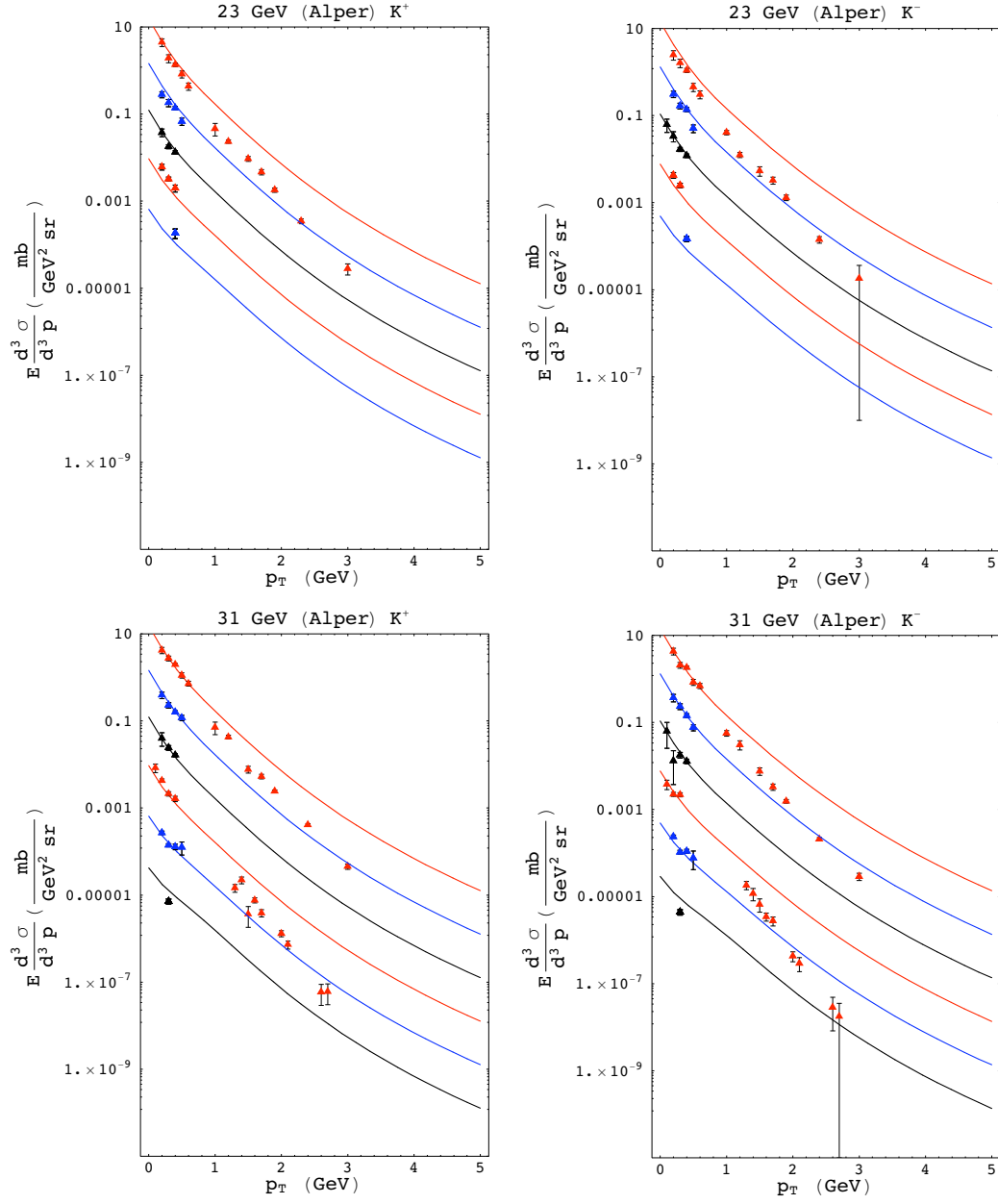


Figure 18: Same as figure 15, except with Alper parameterization.

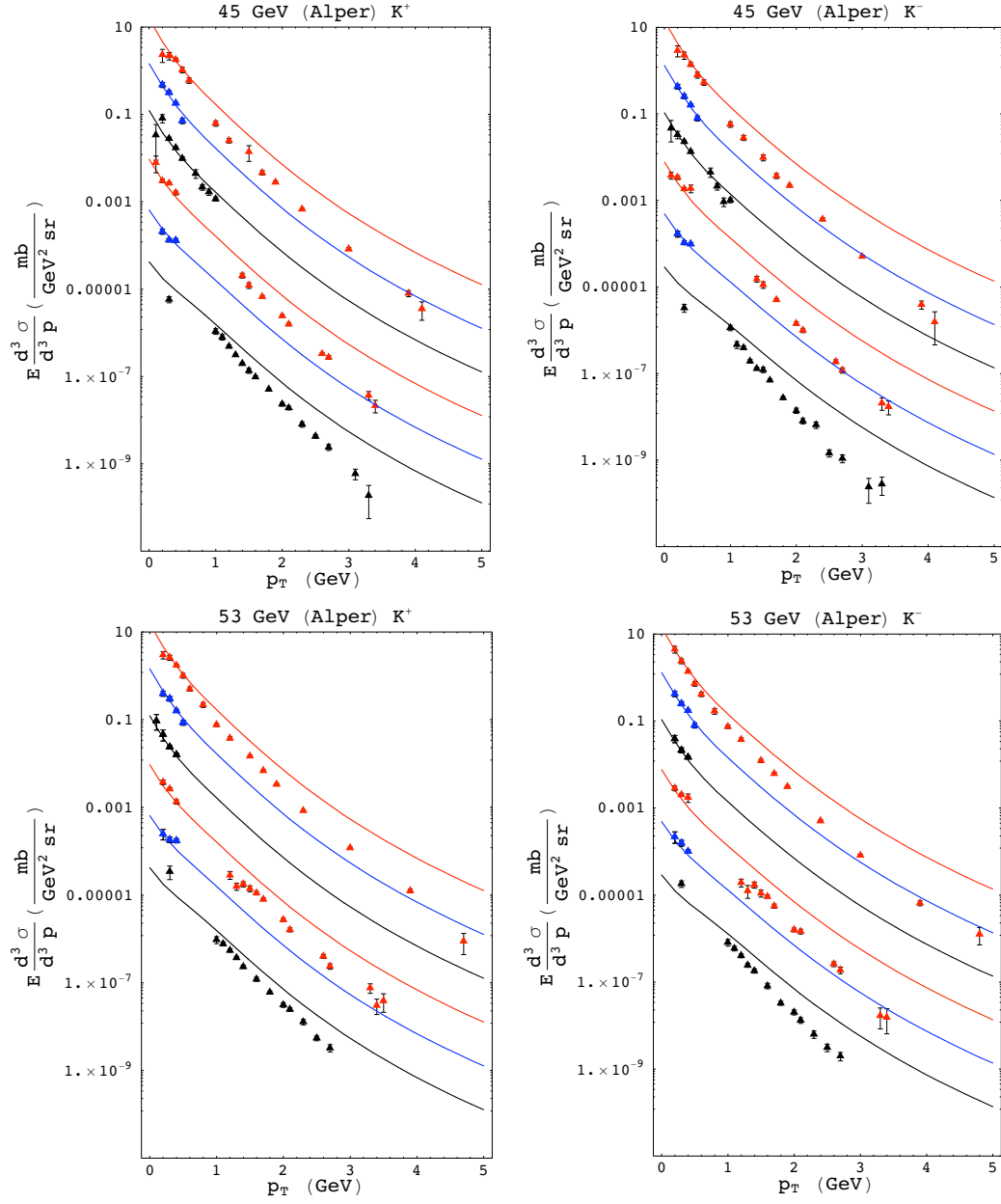


Figure 19: Same as figure 16, except with Alper parameterization.

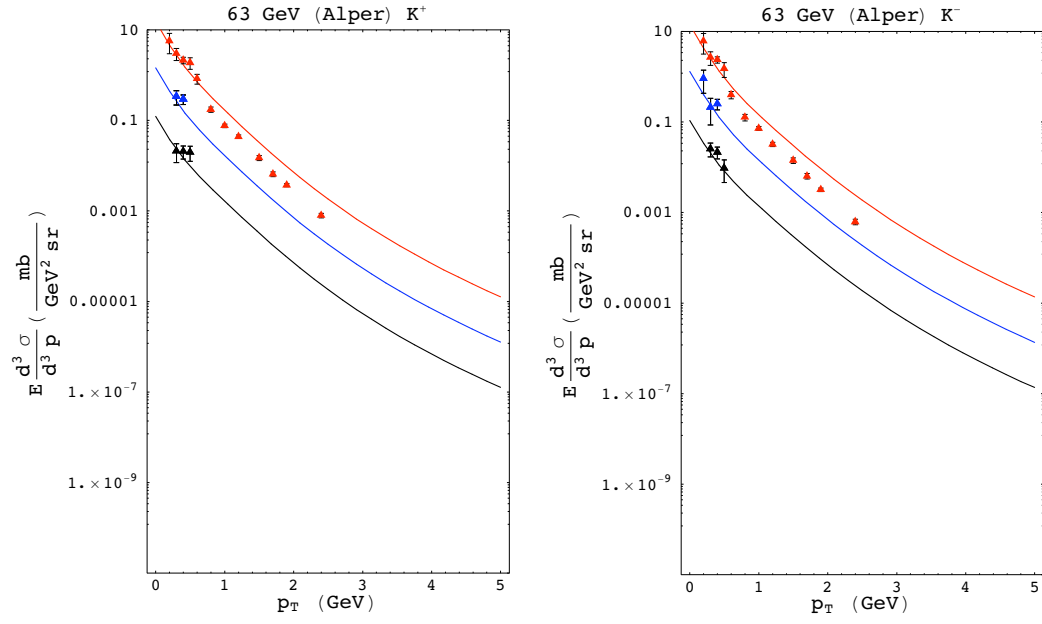


Figure 20: Same as figure 17, except with Alper parameterization.

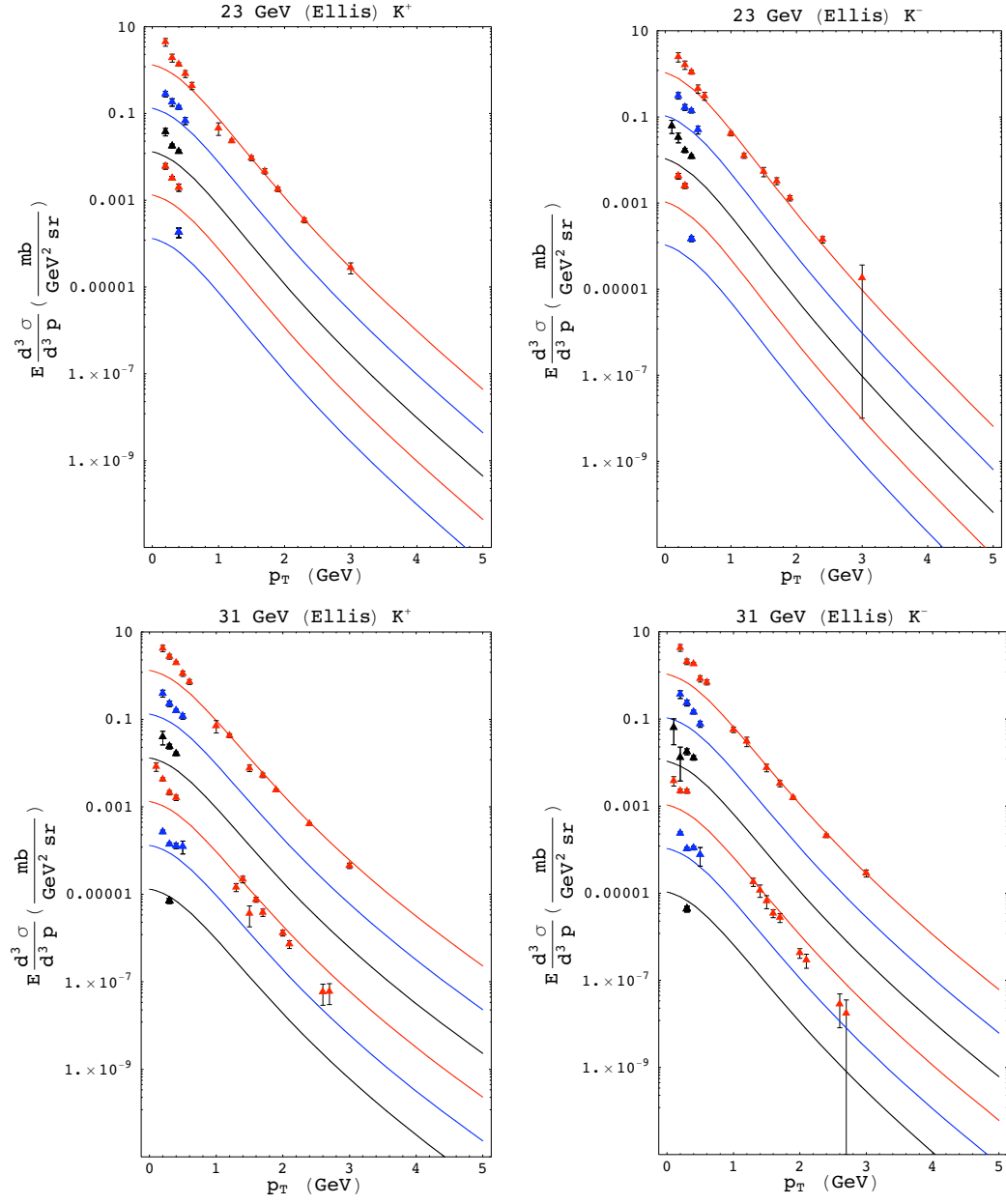


Figure 21: Same as figure 15, except with Ellis parameterization.

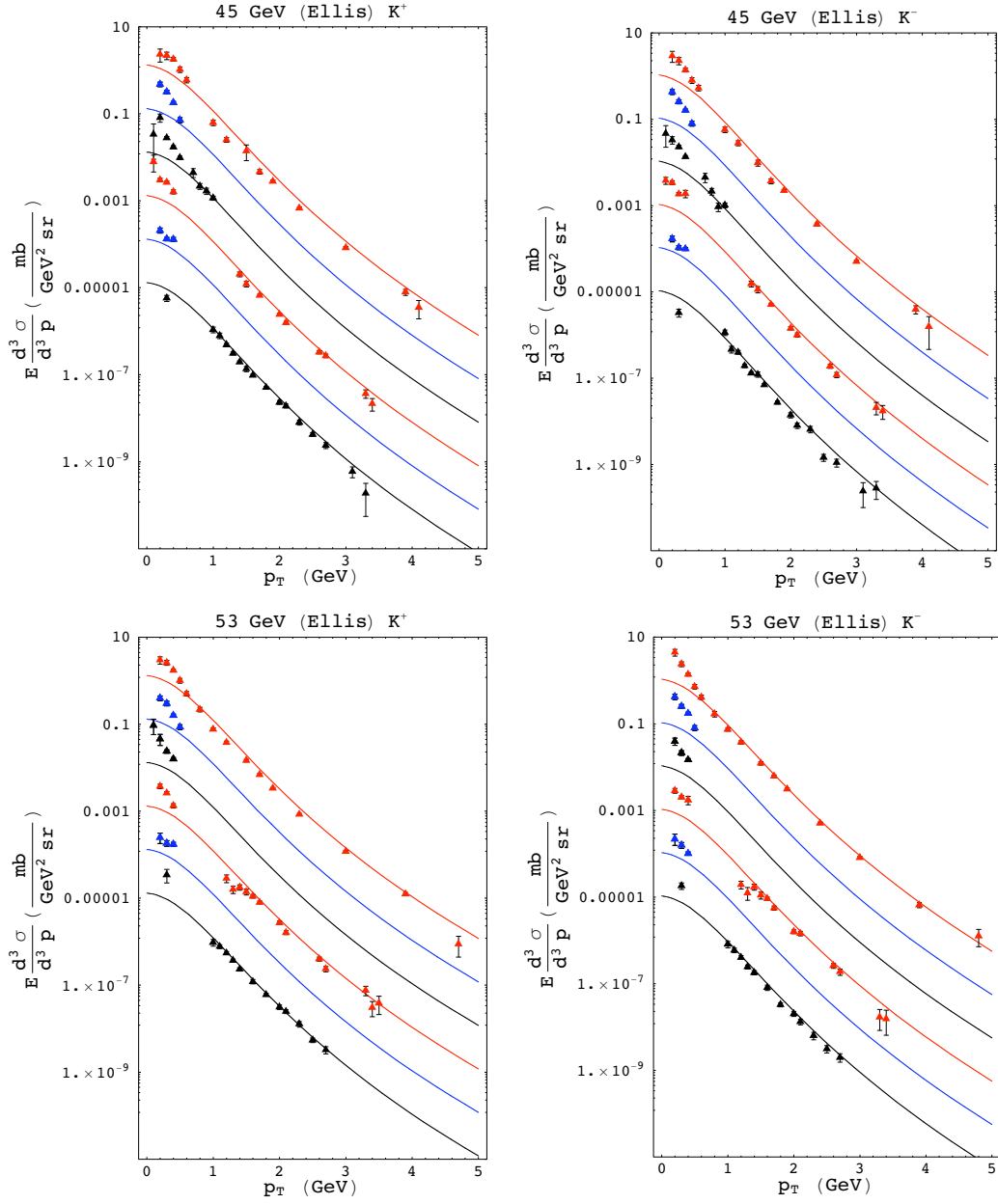


Figure 22: Same as figure 16, except with Ellis parameterization.

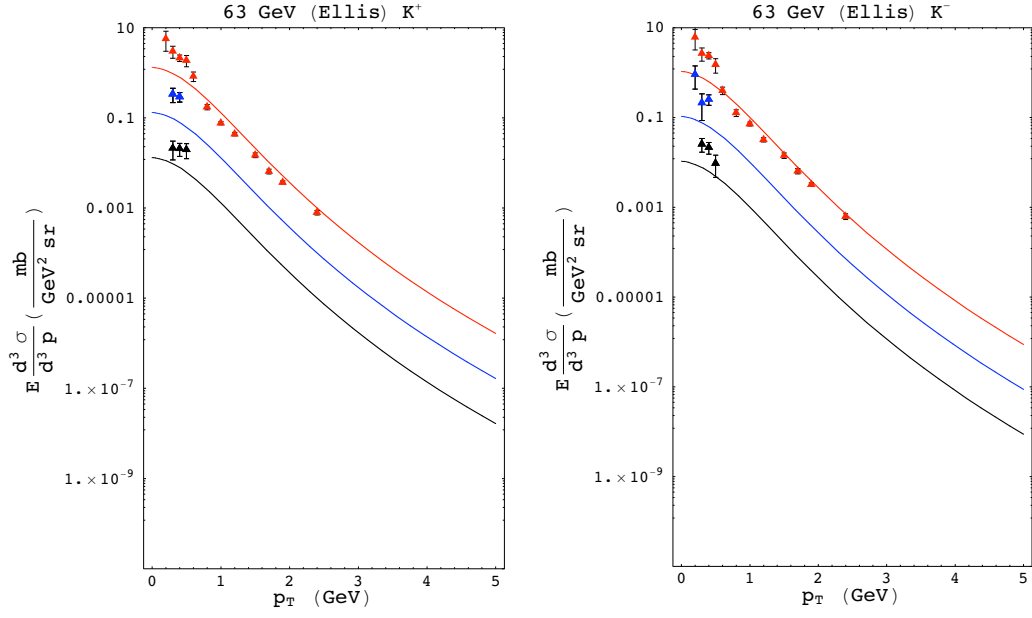


Figure 23: Same as figure 17, except with Ellis parameterization.



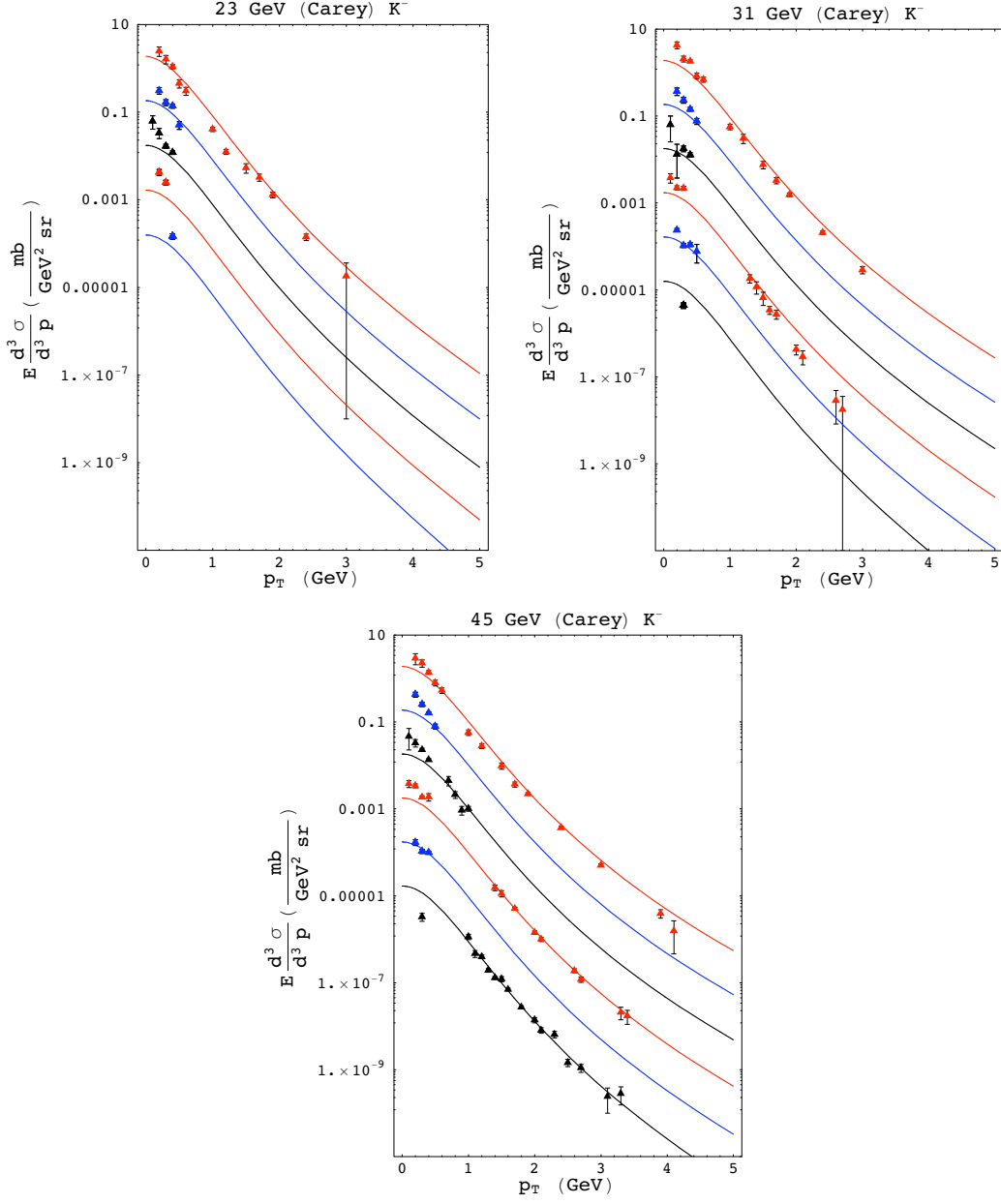


Figure 24: Same as figure 15, except with Carey parameterization with  $\sqrt{s} = 23$  GeV and  $\sqrt{s} = 31$  GeV and  $\sqrt{s} = 45$  GeV for  $K^-$  production only.

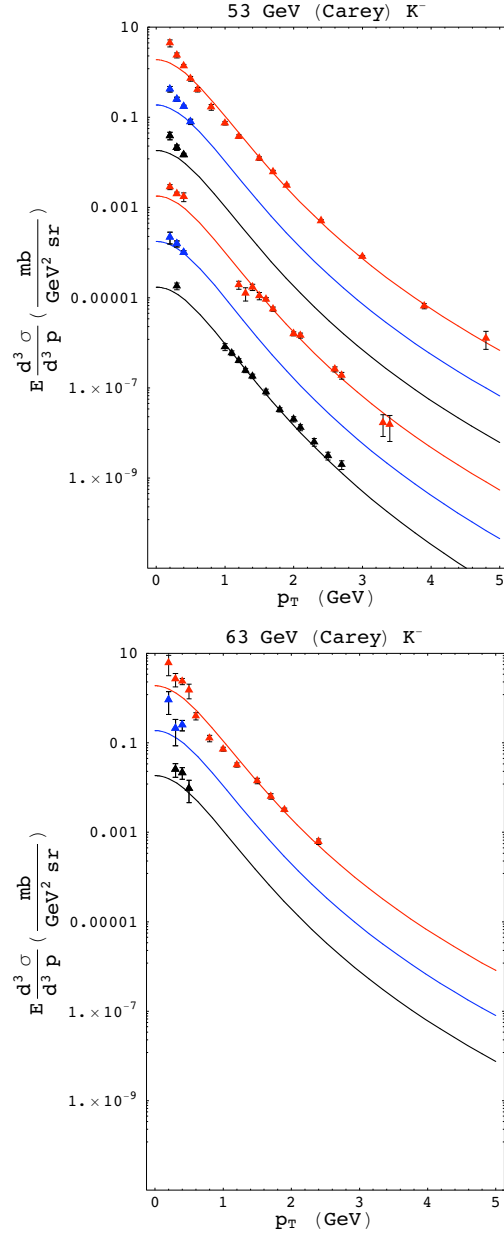


Figure 25: Same as figure 15, except with Carey parameterization with  $\sqrt{s} = 53$  GeV and  $\sqrt{s} = 63$  GeV for  $K^-$  production only.

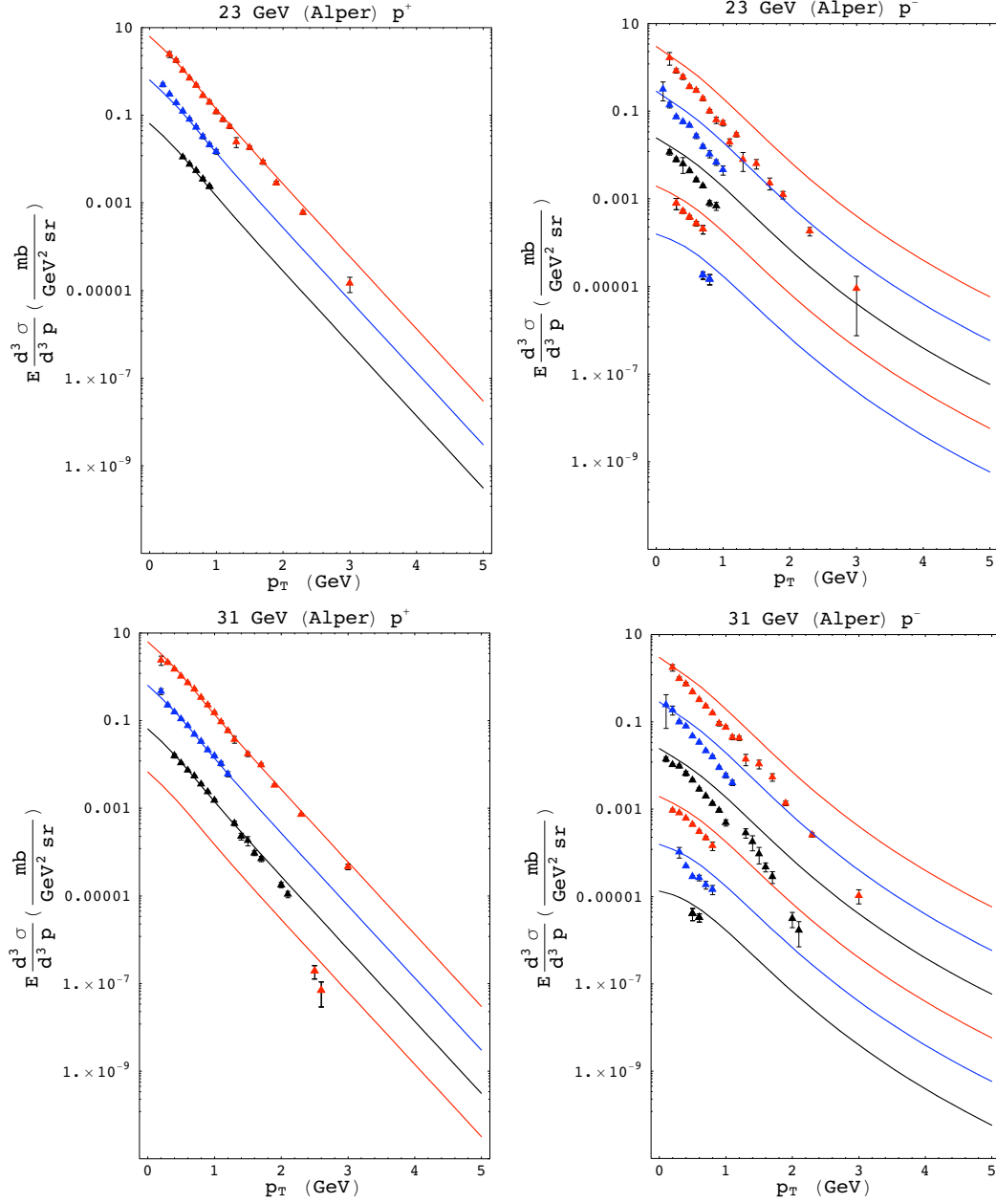


Figure 26: Alper parameterization versus experiment [19] for inclusive proton and antiproton production in  $pp$  collisions at  $\sqrt{s} = 23$  GeV and  $\sqrt{s} = 31$  GeV. The rapidity for all of the top curves in each frame is  $y = 0.0$ . It increases in steps of 0.2 from the top to the bottom curves in each frame. Data and lines in each frame are multiplied successively by 0.1 to allow for a better separation.

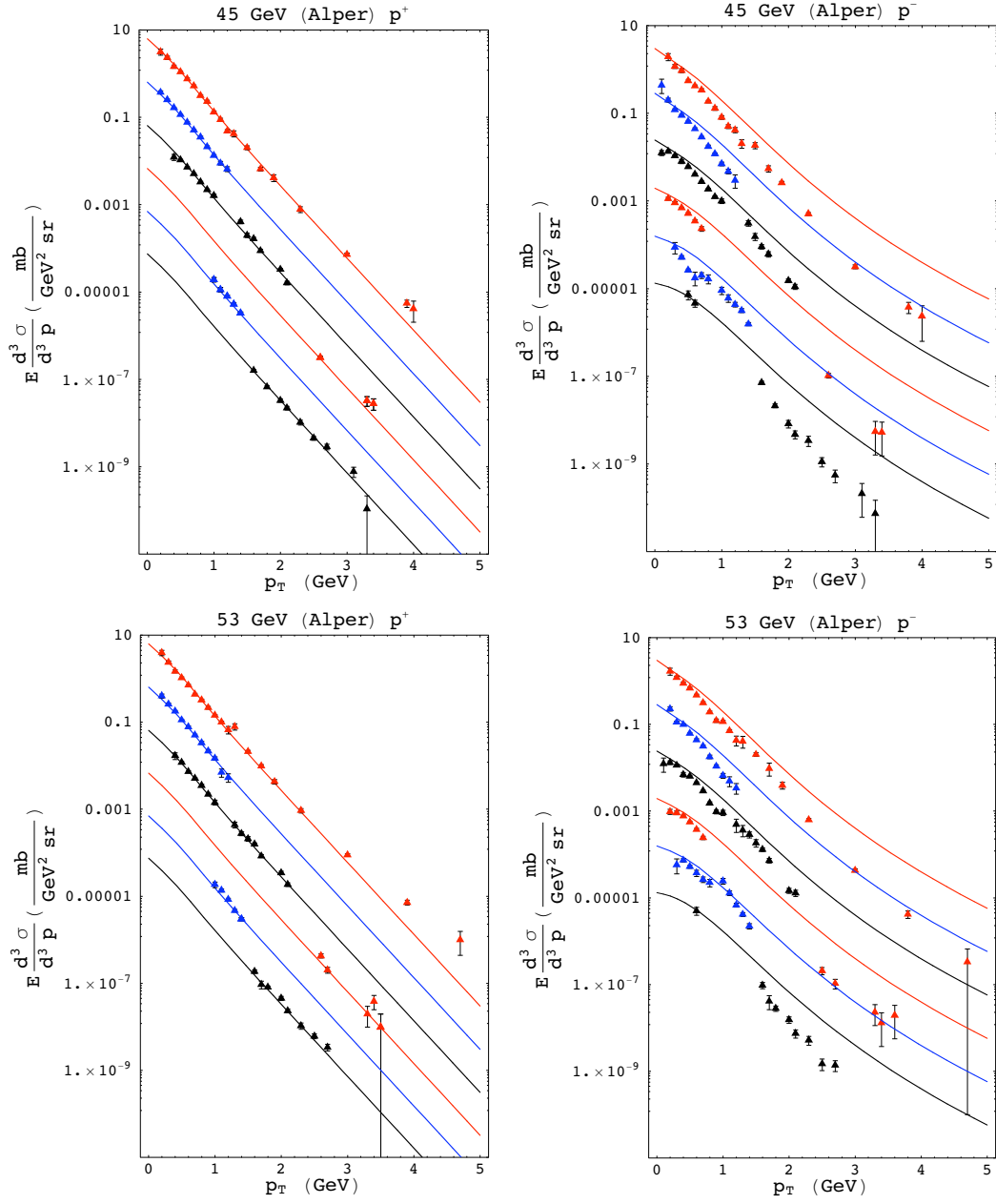


Figure 27: Same as figure 26, except that  $\sqrt{s} = 45$  GeV and  $\sqrt{s} = 53$  GeV.

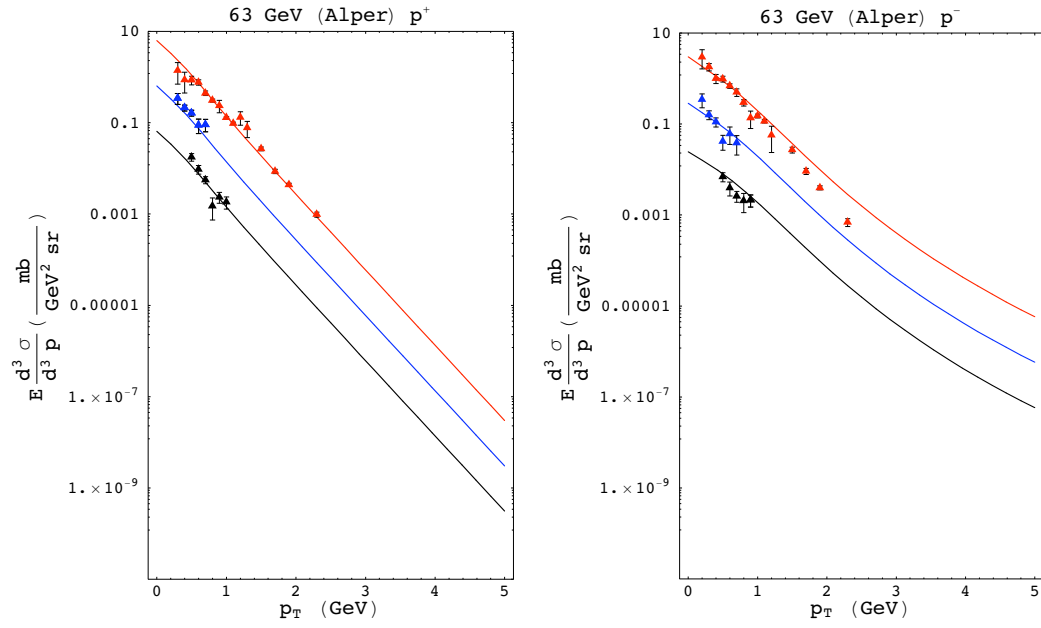


Figure 28: Same as figure 26, except that  $\sqrt{s} = 63 \text{ GeV}$ .

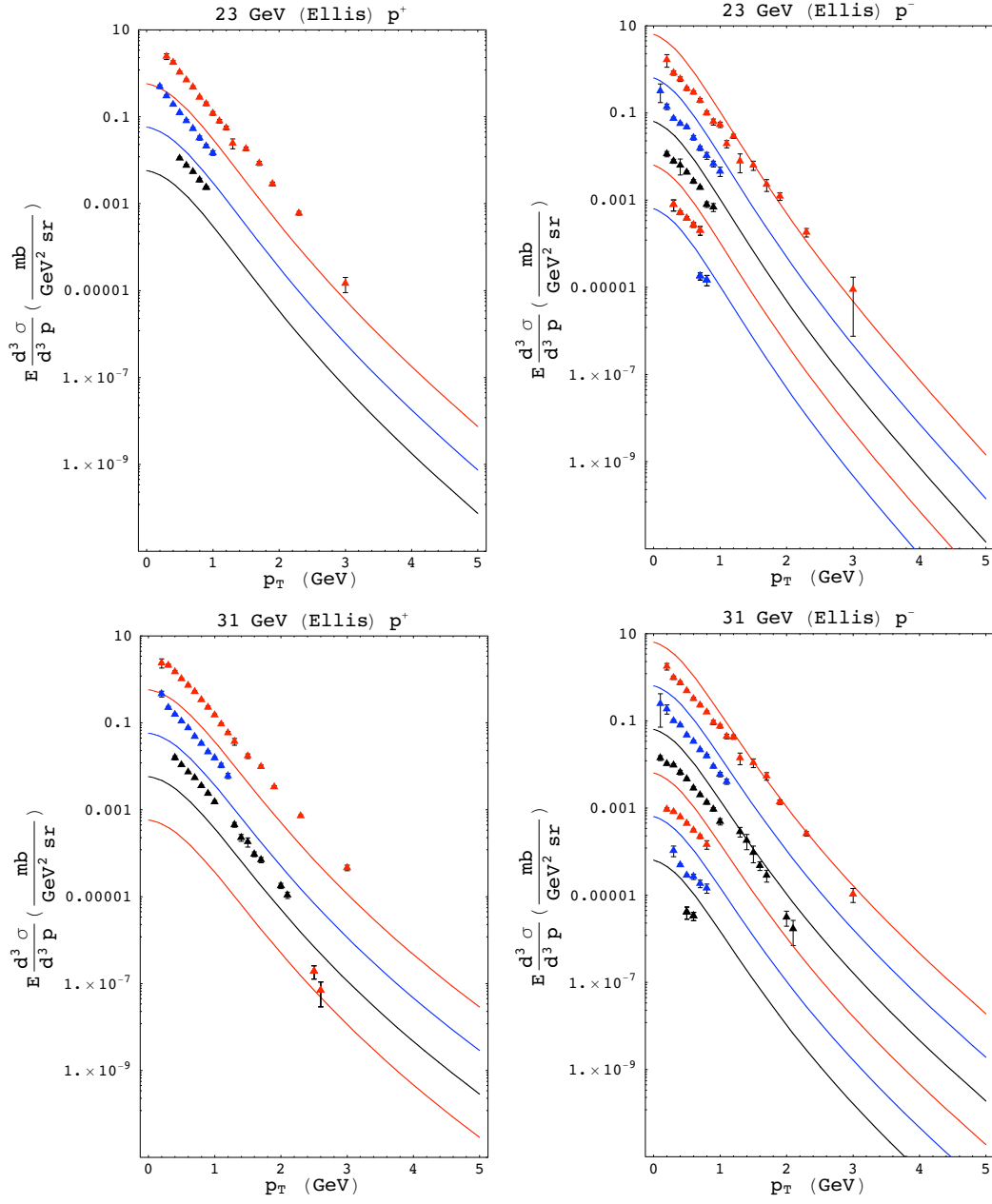


Figure 29: Same as figure 26, except with Ellis parameterization.

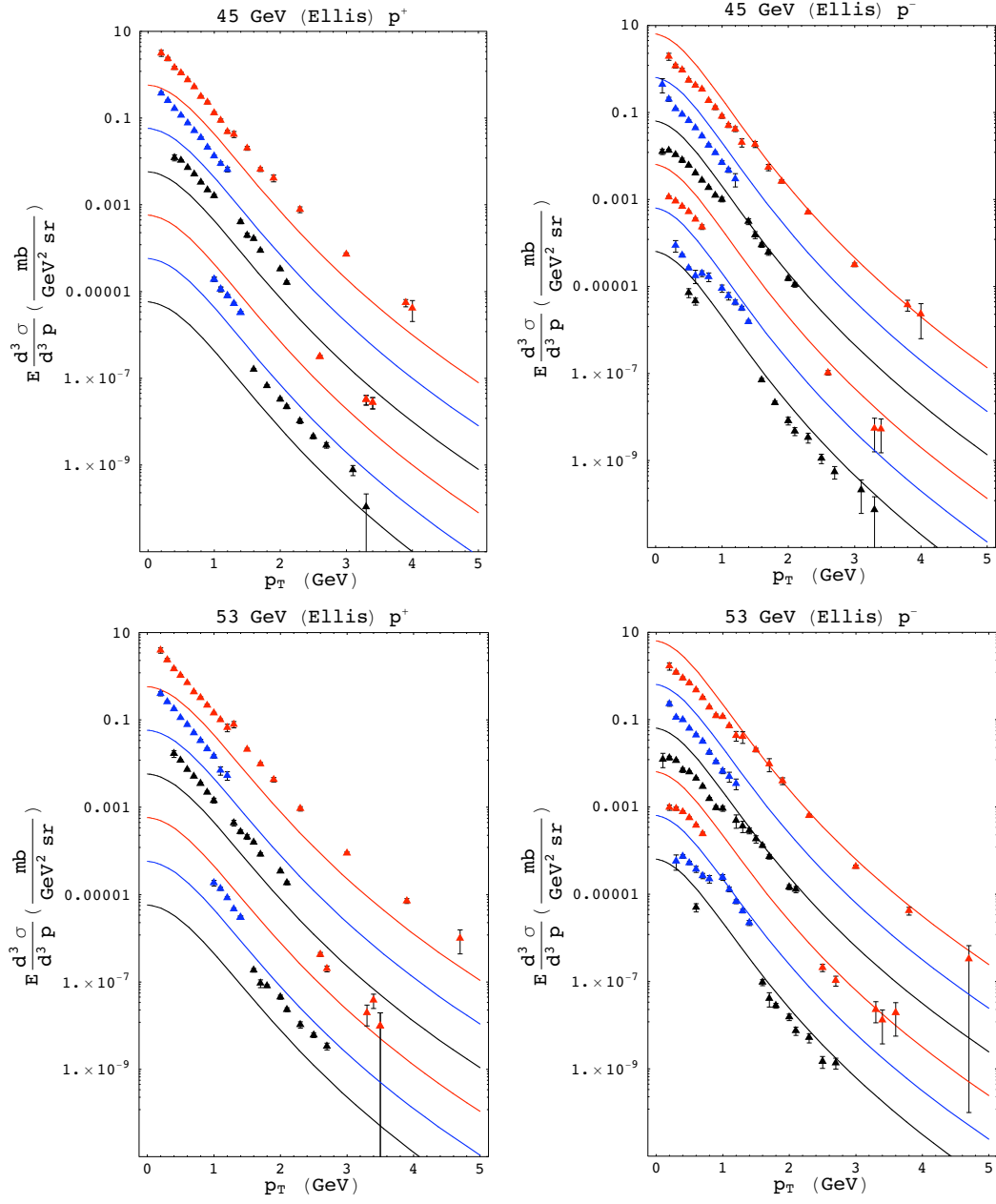


Figure 30: Same as figure 27, except with Ellis parameterization.

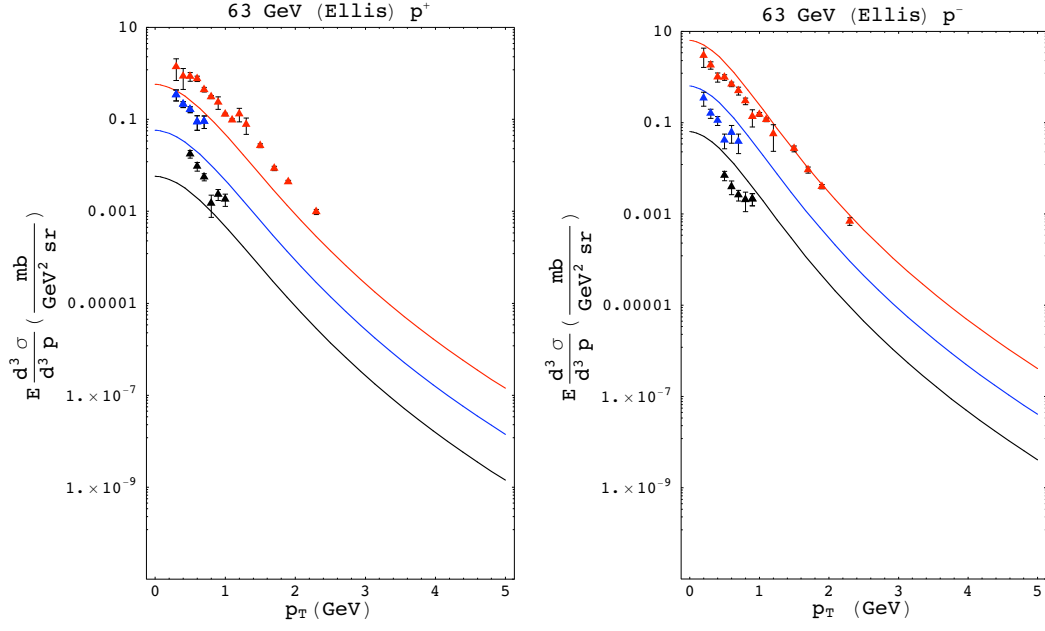


Figure 31: Same as figure 28, except with Ellis parameterization.



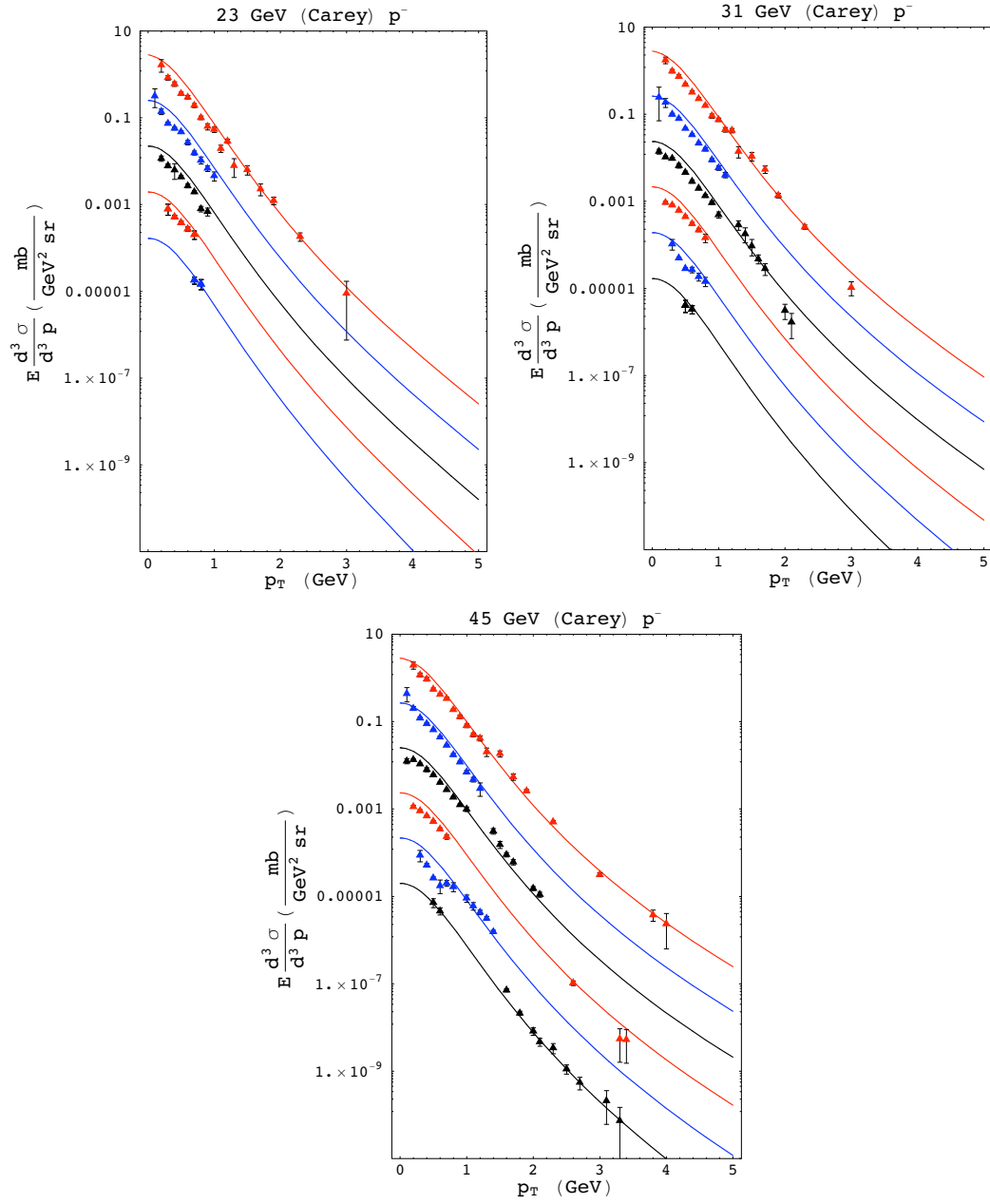


Figure 32: Same as figure 26, except with Carey parameterization with  $\sqrt{s} = 23$  GeV and  $\sqrt{s} = 31$  GeV and  $\sqrt{s} = 45$  GeV for antiproton production only.

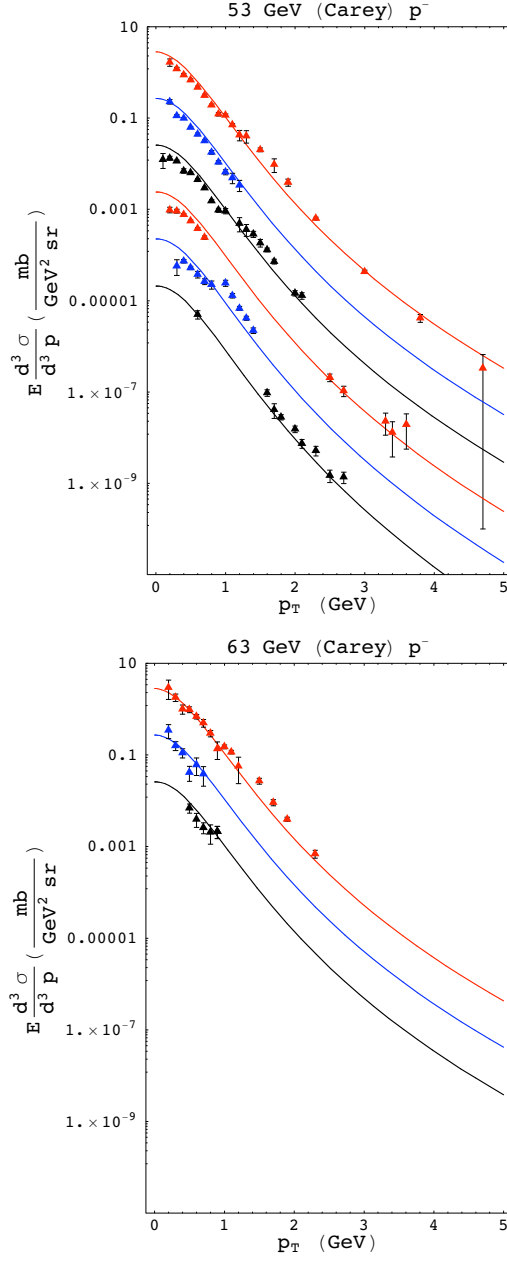


Figure 33: Same as figure 26, except with Carey parameterization with  $\sqrt{s} = 53$  GeV and  $\sqrt{s} = 63$  GeV for antiproton production only.

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